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From Legacy to the Future: Incentivising Demand Migration through Access Fees *

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Abstract

In this paper, we analyze how wholesale access fees of a crucial input can be utilized to influence demands for products of different technologies and the deployment sequence between an incumbent and entrant firm. In a setting of multi-product competition with horizontally differentiated products we find that the access fee gives rise to asymmetric pricing incentives for the entrant firm if she offers a legacy and new product in parallel. The entrant’s price for the new product decreases in the access fee while its legacy price increases with the aim to induce intra-brand legacy-to-new migration of demand. Furthermore, a regulator can depart from the socially optimal access fee and use this entrant’s pricing channel to effectively promote demand side take-up of the new technology. Lastly, it is welfare beneficial in a sequential deployment process, that the entrant moves first to introduce the new technology while such a move can be fostered by a strategic use of the access fee that lowers profits from competition based on legacy products.

JEL Classification: L13, L51, L96, D4.

Keywords: Access pricing, Multi-product competition, Product differentiation, Next generation networks.

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1 Introduction

In network industries or technology driven markets in general, innovative technologies enable new products to consumers and transform the competitive environment for suppliers. Especially in the context of telecommunications markets the transition from old, often-copper based, network infrastructure towards a new future-proof (fiber-)technology is currently ongoing. However, deployment of the new infrastructure and making associated products more widely available is costly and a fast take-up in demand is not only a desire by policymakers but also operators who seek a return on their deployment investments. Since consumers’ demand is only slowly migrating away from legacy products to new ones, European and national authorities such as the European Parliament (2018) are keen on promoting this demand migration where deployment has already taken place.

Among the regulatory toolkit already applied to telecommunications markets are access prices which are paid by an entrant operator to an incumbent firm. The trade-off between static- and dynamic efficiency resulting from those fees when considering their effect on deployment of new technologies is well researched.\(^1\) However, the interplay of those access fees and the sequential deployment path between an entrant and an incumbent firm, as well as implications for demand (take-up) steering has been largely ignored by the recent literature. Therefore, the main question this paper addresses is the following: Can wholesale access fees be used to promote demand for new technologies’ products and influence the specific deployment sequence between an incumbent and an entrant firm?

To answer this, we study a setting of competition between multi-product incumbent and entrant firms who both offer up to two horizontally differentiated products based on distinct technologies. While most approaches in this field model an immediate replacement of the legacy products by the new technology,\(^2\), in our setting both technologies are operated in parallel since deployment paths in reality imply a coexistence of different technologies and prolonged transition phases (Bourreau et al., 2012). For the investigation of differentiated deployment paths, we separate the analysis into distinct scenarios with varying configurations of available products similar to Bourreau et al. (2014). These either characterize temporal steps in an evolutionary deployment process or can be interpreted as geographic areas with varying deployment progress.

We produce the following main results: First, the wholesale access fee generally increases consumer prices of the incumbent while it gives rise to asymmetric pricing incentives for the entrant firm if she offers a legacy and new product in parallel. The entrant’s price for the new product reduces in the access fee while its legacy price increases with the aim to induce intra-brand legacy-to-new migration of demand. Second, a regulator

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\(^{1}\)Valletti (2003) and Cambini and Jiang (2009) offer a profound literature review on the interplay of regulatory measures and investments. For a specific context in the field of FttH deployments, see Flacher and Jennquin (2014).

can depart from the socially optimal access fee and use this entrant’s pricing channel to effectively promote demand side take-up of the new technology. Lastly, it is welfare beneficial in a sequential deployment process, that the entrant moves first to introduce the new technology. Furthermore, the access fee can be utilized to motivate such a first move by the entrant through lowering profits from competition based on legacy products.

Our results mainly contribute to three streams of literature. The first one is the extensive body on supply- and to a lesser degree demand-migration from an old technology to a new one (i). Second, the literature on cost parameters and access fees within oligopoly competition (ii) and, finally, models of horizontal differentiation with non-localized spatial competition (iii).

The majority of studies in the field of technology advancement from old to new (i) considers an immediate switch-off of the legacy products once the new technology is introduced (Gans and Williams, 1999; Foros, 2004; Hori and Mizuno, 2006; Jeanjean and Liang, 2012; Tselekounis et al., 2014). However, products based on the old legacy technology are regularly not discontinued immediately but rather phased out if demand for those diminish after a period of time or they have become non-profitable. Recent studies which depart from this feature and instead allow for a co-existence of legacy and new technologies are Bourreau et al. (2012) and Bourreau et al. (2014). Similar to our result both find that a higher access fee for the legacy technology increases the entrant’s incentives to invest in the new technology. The effect on incumbent’s profits and investment incentives, however, is ambiguous due to two counteracting effects which are coined by the authors as the “wholesale revenue effect” and the “retail migration effect”. Bourreau et al. (2014) is especially related to our study as their approach includes also distinct scenarios as characterizations of regional differences. However, they exogenously determine the sequence of deployment moves and assume that the incumbent will invest first in the new technology. In reality this is not always the case and our results show, that an entrant’s first move is not only the welfare optimal path but also gives rise to additional steering potential of new technology take-up.³

Given that we investigate the effect of an access fee within oligopoly competition, we contribute to studies in this field (ii). Our result of a generally positive pass-through of the access fee to product prices of both firms can be traced back to early findings of Dixit (1986). Although the access fee is only paid by the entrant, it is also perceived by the receiving incumbent as opportunity cost parameter. If the incumbent increases end consumer sales, she does so at the expense of wholesale sales. This directly characterizes the previously mentioned “wholesale revenue effect” by Bourreau et al. (2014) which materializes also in our setting.⁴ The incumbent’s trade-off in the access price has been

³Other features of Bourreau et al. (2014) are that different technologies may indeed coexist, but only between firms. Once a firm invests in the new technology, it immediately discontinues the legacy products. This does not allow for intra-brand demand migration which is especially the driving force in our setting. Furthermore, the authors abstract from any heterogeneous consumer preferences with respect to suppliers.

⁴We do not model an own production option for the entrant to avoid the wholesale access fee. Sappington (2005) shows that the entrant would buy an incumbent’s upstream access at any price below its own cost to
formalized by Mandy (2009) and Gayle and Weisman (2007) for products as strategic substitutes and single-product firms facing linear demands. These features apply to three of our four scenario settings as well (all but Scenario II) and we reproduce their main finding of an entrant’s profit function which is decreasing in the access fee. Furthermore, our approach extends the literature in this context in that we investigate competition among multi-product firms and also account for heterogeneous consumer preferences.

To do so, we employ a variation of the Spokes model put forward by Chen and Riodan (2007) within the group of spatial non-localized competition frameworks (iii). The plethora of contributions in this field formalize the concept of monopolistic competition by Chamberlin (1949). While the Spokes model is no exception in this regard, it derives from Hart (1985) and applies to a spatial setting that exhibits features distinct from other multi-product frameworks such as Salop (1979). The most important of which is that each firm (or product) simultaneously competes against all other alternatives although a single consumer has only preferences for a subset of options (Wolinsky, 1986). The Spokes model was first applied to multi-product competition in connection with access fees by Brito and Tselekounis (2017) who study the effect of those fees to an entrant’s profit function. The authors employ a similar setup in which two firms both offer a legacy and a new product in parallel. They find that the access fee may affect the entrant’s profits positively, which we also reproduce in our Scenario II when all consumer preferences are matched and aggregate output is fixed. However, Brito and Tselekounis (2017) fail to characterize that this result hinges crucially on the full coverage of consumer preferences and perfectly inelastic aggregate demand. We show in other scenarios that parameter domains generally rule out this positive relation between the access fee and entrant’s profits due to demand elasticities being non-zero. Hence, it is precisely our Scenario comparison of different available product configurations that separates our study from previous approaches in this field.

The remainder of the paper is structured as follows. Section 2 describes the formal model and Section 3 characterizes private and social equilibrium outcomes in the different scenarios. Based on these results, Section 4 elaborates on the potential to influence aggregate demand for the new technology via the access fee while Section 5 identifies the welfare optimal deployment sequence and how to promote it. Finally, Section 6 concludes.

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5 Another feature is that total output is not necessarily fixed to unity but rather depends on realized equilibrium prices. This is a core feature in our Scenario analysis that exhibit consumer preferences which are not matched by available products. This implies demand elasticities to be positive but never infinite, irrespective of how many firms or products are being offered. This stands in contrast to Salop (1979) and Perloff and Salop (1985) in which the distance between infinitely many firms approaches zero and elasticity of demand reaches infinite in the limit.

6 See also Mandy (2009) who show this generally for strategic substitutes and downward sloping linear demands.

7 Additionally, Brito and Tselekounis (2017) restrict consumer preferences to be either firm- or technology-specific. This assumption is rather restrictive since it implicitly rules out consumers who consider between products that differ in both the firm and technology dimension. We relax this assumption as it would limit channels of demand migration a-priori and stands in stark contrast to our research objective.
2 The Model

Our main model relies on a modified version of Chen and Riordan (2007) model on competition with horizontal differentiation across spokes. In this spatial model, consumers’ preferences are distributed across multiple $N$ spokes across the preference space. If $N = 2$ and both spokes are served by two competing firms, the Spokes model collapses to the standard Hotelling model. In our model extension we consider that consumers’ preferences are distributed across $N = 4$ spokes and resemble internet access products configuration of differing suppliers and access technology. Additionally, we analyze different deployment scenarios which highlight varying amounts of product configurations being offered such that we can produce insights on the interplay of the following real market characteristics.

1) Deployment and operation of the higher quality access technology (fiber) will be carried out alongside an inferior access technology (V-DSL) which will not be switched off immediately.

2) Deployment processes take time and can be sequentially asymmetric between incumbent and entrant firms.

3) The demand/adoption of the higher quality product is (potentially) affected by a (regulated) wholesale access fee for the legacy copper network that is paid by the entrant to the incumbent.

We separate our theoretical analysis in 4 scenarios that each resemble a different evolutionary step in the deployment process of a higher quality technology. We assume that preferences have already formed for each of the four possible product configurations, that is, for each available combination of firm $j$ and access technology $k$, and that these are common to all scenarios. However, the deployment of network infrastructure does change across the scenarios and, hence, does product availability.

In each scenario there is competition between $K = 2$ firms. A firm $j$ takes either the role of an incumbent $j = I$ or that of an entrant firm $j = E$. Given the scenario, the amount of available product configurations $n = \{2, 3, 4\}$ changes and the technology $k$ of a single product $jk$ resembles either a standard connection $k = L$ or a higher quality one $k = H$. Hence, each product $jk$ is perfectly identified. Scenario I and II reflect in this notion the start and end-point of an evolutionary deployment process of a higher quality access technology starting with scenario I in which only the standard connection is available ($n = 2; j = \{I, E\}; k = L$) and ending with scenario II where the higher quality technology is fully deployed by both firms ($n = 4; j = \{I, E\}; k = \{L, H\}$). Figure 1 visualizes the preference space and product availability in these scenarios.

While start and ending phase of a deployment process are clear the path to a complete roll-out of the newer technology can be different. We consider two possible deployment paths that differ in the sequence of the firm’s moves. In scenario III the incumbent moves
first and introduces a product based on the new technology \((jk = IH)\) in parallel to its own and the entrant’s standard quality. In scenario IV the situation is reversed and the entrant rolls-out the new technology first and offers the associated product \((jk = EH)\). Figure 2 displays the preference space and product availability in these scenarios accordingly.

Although the amount of available products \(n\) differ among the scenarios, other model characteristics do not. Also the analysis in each scenario follows a clear pattern to strengthen the comparability of the results. Both of which are described in the following.

Regardless of the scenario, competition is always between two firms an incumbent and an entrant. Both of which always offer a product of basic connection to consumers. The associated legacy network is operated by the incumbent and unlike to the analysis of Brito and Tselekounis (2017) in which access to the legacy network is possible at zero costs for both firms, we consider that it is accessible to the entrant at an access charge. This access charge \(w\), with \(w > 0\), is a positive per-unit payment paid by the entrant to the incumbent dependent on the realized demand for the entrant’s basic access product.
Additionally, we restrict \( w \) such that it may not exceed the entrant’s selling price of its basic product \( (w < p_{EL}) \) and hence its profit margin remains non-negative. We are confident that we capture with this the core features of wholesale access fees as part of LLU from real world scenarios reasonably well. Furthermore, we abstract from any prior deployment costs which can be considered to be sunk and are not decision relevant for the following competitive behavior.

Firms compete in prices \( p_{jk} \), with \( j \in \{I, E\}, k \in \{H, L\} \) and we assume that firms maximize horizontal product heterogeneity and, thus, the locations of a product \( jk \) is fixed to the end-point of its respective spoke in the preference space. Apart from the pricing decision the incumbent firm sets also \( w \) as strategic variable. Apart from \( w \), which is paid by the entrant, we abstract from any other cost parameter and fix marginal costs of serving a consumer to be zero.

Common to all scenarios is that consumer mass is normalized to unity and their respective product preferences are uniformly distributed across the whole preference space, that is, all \( N = 4 \) spokes (see Figures 1 & 2). Hence, we look at market situations in which preferences also for the higher quality products have already been formed by consumers, but available products that match those are not necessarily being offered yet. We follow the standard configuration of Chen and Riordan (2007) and assign spatial addresses to the spoke endpoints of either 0 or 1, which consequences the midpoint of all spokes to be located at \( x^M = 0.5 \). A respective consumer’s location in the preference space is determined by a vector \((l_{jk}, x_{jk})\), where \( l_{jk} \) is the spoke the consumer is located on and \( x_{jk} \) represents the distance \( \Delta_{jk} \) to the product variety of \( jk \), that is, the endpoint of the spoke \( l_{jk} \). Given that preferences for all other products are symmetric, the spatial distance for any consumer \((l_{jk}, x_{jk})\) to an alternative product \( j'k' \), \( j' \neq j \), \( k' \neq k \), is determined by \( \Delta_{j'k'} = 1 - x_{jk} \) and goes through the midpoint \( x^M \). Purchasing a product that does not perfectly match a consumer’s product preference involves positive and linear transportation costs of \( t, t > 0 \). Since travel distance is lowest, product \( jk \) is consumer \((l_{jk}, x_{jk})\)’s first preferred product option. Each consumer also has a second preferred option when making a purchasing decision. This second preferred product can be any \( j'k' \) of the remaining \( N - 1 \) options in the preference space, which is determined by nature’s draw with probability \( \frac{1}{N-1} \).

If a consumer purchases either her first or second preference she realizes a base utility of \( v \), which can be interpreted as the benefit of having a generally fast internet access (NGN) irrespective of the underlying technology. However, dependent on the scenario, a consumer’s first or second preference (or both) may not be matched by the available products and she faces the decision of purchasing a NGN product or no product at all. Her utility in the case of no purchase is normalized to zero without loss of generality. This can be interpreted as choosing a numeraire good which only provides the most essential internet connectivity (e-mail, web-browsing, etc.) and is not part of next-generation access.
technologies. 8

Additionally, we also introduce a technology specific quality parameter $\delta_k$ to account for the higher quality technology in contrast to horizontal preferences. Without loss of generality, we normalize $\delta_L = 0$ and $\delta_H = \delta$, such that a consumer who purchases a superior technology’s product receives a positive utility premium of $\delta$, with $\delta > 0$, as the incremental quality advantage compared to a basic NGN access. 9 Furthermore, we restrict $\delta < 3t$ to ensure existence of interior solutions for the equilibria of the model. This implies that the incremental quality advantage of of the newer technology tariffs may not be arbitrarily large in comparison to consumers’ inherent product preferences. Finally, consumers pay a product price of $p_{jk}$ such that total utility is determined as follows.

$$U_{l_{jk}, x_{jk}, j'k'} = \begin{cases} 
 v + \delta_k - t \cdot x_{jk} - p_{jk} & \text{if purchasing product } jk, \\
 v + \delta_k - t \cdot (1 - x_{jk}) - p_{j'k'} & \text{if purchasing product } j'k', \\
 0 & \text{otherwise.}
\end{cases}$$  \hspace{1cm} (1)

Based on the respective scenario, there may be three relevant consumer groups that together form the demand for a given product $jk$. First, consumers for whom both first and second preferences are part of the available products, that is, $jk, j'k' \in \{2, \ldots, n\}$. Second, whose first preference is $jk$ and whose second is not available, and lastly, those whose first preference is not available but $jk$ as the second preferred is ($jk \in \{2, \ldots, n\}$ and $j'k' \notin \{2, \ldots, n\}$).

For consumers whose both preferences are available, the location of the consumer $(l_{jk}, x_{jk}, j'k')$ who is indifferent between her first preferred product option $jk$ and another randomly chosen second alternative $j'k'$ is given by Equation 2.

$$l_{jk}, x_{jk}, j'k') = \max \left\{ \min \left\{ \frac{1}{2} + \frac{(p_{j'k'} - p_{jk}) + (\delta_k - \delta_{k'})}{2t}, 1 \right\}, 0 \right\} \hspace{1cm} (2)$$

The mass of such consumers who purchase the product $jk$ is then given by

$$q_{jk} = 2 \frac{1}{N} \frac{1}{N - 1} \sum_{j'k' \neq jk, j'k' \in \{1, \ldots, K\}} \max \left\{ \min \left\{ \frac{1}{2} + \frac{(p_{j'k'} - p_{jk}) + (\delta_k - \delta_{k'})}{2t}, 1 \right\}, 0 \right\} \hspace{1cm} (3)$$

where $2/N$ is the density of consumers located on the two spokes in consideration $l_{jk}$ and $l_{j'k'}$.

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8In the context of internet access it is reasonable to assume that each consumer will only purchase exactly one unit.

9With technology specificity of $\delta_k$ we mean that consumers have no differentiated valuation between an incumbent’s or an entrant’s product of the identical technology. This assumption is justified by the fact that the entrant has to buy access to the legacy network of the incumbent and therefore the quality of the underlying network is identical for customers of both companies. Furthermore, we abstract from any form of strategic quality degradation or sabotage by the incumbent.
Consumers part of the second and third group, for which product $jk$ is the only available from their two preferences, prefer to purchase $jk$ if $v \geq p_{jk} + t \cdot x_{jk} - \delta_k$ ($jk$ first preference) or if $v \geq p_{jk} + t \cdot (1-x_{jk}) - \delta_k$ ($jk$ second preference). Hence, the indifferent consumer in these two segments is just indifferent between a purchase of $jk$ or choose the zero utility outside option of the non-NGN numeraire good. The aggregate demand for $jk$ from these second and third group consumers can be derived to be the following.

$$q''_{jk} = \frac{2}{N} \frac{N-n}{N-1} \max \{ \min \{ v - p_{jk} - t \cdot x_{jk} + \delta_k, 1 \}, 0 \}$$  \hspace{1cm} (4)

Summing up these categories of consumers from (3) and (4), the entire demand for a product $jk$ is given by

$$Q''_{jk} = q'_{jk} + q''_{jk}.$$  \hspace{1cm} (5)

Correspondingly to this, consumer surplus in each scenario is given by the aggregate utility received by all consumers who purchase any of the products available lowered by the individual transportation costs and prices paid. This is determined by the following:

$$CS = \sum_{j' \neq j, j' \in \{1, \ldots, K\}} \frac{2}{N} \frac{1}{N-1} \int_0^{(l_{jk} x_{jk}, j'k')} v + \delta_k - t \cdot x_{jk} - p_{jk} \, dx$$  \hspace{1cm} (6)

Given that available products differ across scenarios, firm profit functions do as well. Hence, we postpone these to the scenario sections and conclude the general model framework at this point. The analyses in the following sections are following a clear pattern in that profit functions are being discussed and equilibrium solutions to the private optimization problem are derived. These private solutions are then compared to the socially optimal level a welfare maximizing social planner (or regulator) would choose and welfare implications are being discussed.

The timing of actions, however, is identical across all scenarios. First, $w$ is set either privately by the incumbent or by a social planner. Subsequently, firms choose their prices and , finally, consumers make their purchasing decision and demands realize. This sequence of strategic interaction is the most realistic implementation since wholesale access prices are determined for a specific period in advance and are thus common knowledge to all participating agents. Hence, a simultaneous optimization with respect to the access fee $w$ and product prices $p_{jk}$ would be mathematically possible but unrealistic, irrespective of whom is choosing $w$. In the following scenario analyses we solve for Nash-equilibria by applying backward induction.
3 Scenario analyses

Scenario I - Legacy products

In Scenario I competition between the incumbent and the entrant exclusively takes place based on the legacy network infrastructure and associated products. The amount of available product configurations \( n = 2 \) is the most limited in any scenario such that \( j \in \{I, E\}, k = L \). From a real market interpretation, this resembles a situation just prior to the beginning of the deployment process of a new superior technology. In such a situation, consumers’ preferences for these new access products have already formed but are not yet served by any of the offered products. Alternatively, this situation might also be present in rural and sparsely populated areas today, where any investment into new deployments (of fiber) are not lucrative.

Through the full preference space that contains \( N = 4 \) spokes and only \( n = 2 \) product configurations available, half of consumers are located on spokes on which are ‘unserved’, that is, is not populated by a firm’s product configurations. Hence, those consumers’ first preferred product option is not available. If the second preferred option is also not available, consumers immediately drop out of the market for NGN products and choose to "buy" the numeraire good as the zero utility outside option. If the second preferred option however is available, there are marginal consumers who are located on an unserved spoke and are indifferent between purchasing their second preference or the numeraire (zero-utility outside option). Figure 3 visualizes the marginal consumers on unserved spokes and the numeraire drop-out.

![Figure 3: S1 - Preference space, available products and numeraire drop-out](image)

The demand for an available product \( jk \) as second preference from these comparisons
against the numeraire are included in (4). Consequently, this implies that the market is not necessarily covered and total demand for NGN access products may be below 1. Furthermore, products being not available implies that the following calculations are dependent on the base valuation of having NGN access via (4). Naturally, the marginal consumers with first preferences for unavailable products still must lie between 0 and 1, that is, \(0 \leq (l_{jk}, \hat{x}_{jk}, j'k') \leq 1\), which is also expressed by the min-max environment in (2). From this one can infer upper and lower bounds on the parameters of \(v\) and also \(w\) which we denote by \(v_{jk}, \hat{v}_{jk}, w_{jk}, w_{jk}\) and are given by the comparisons of the available product \(jk\) against the zero utility numeraire. 11 It can be shown that the binding restrictions on \(v\) are

\[
v = v_{IL} = 2 + \frac{3w}{11}; \quad v = v_{EL} = \frac{t}{5} + \frac{51w}{55}
\]

and on \(w\) are the following.

\[
w = w_{EL} = \frac{55v}{51} - \frac{11t}{51}; \quad w = \max \left\{ w_{EL} = \frac{55v}{51} - \frac{110t}{51}, w_{IL} = \frac{11v}{3} - \frac{22t}{3} + 0 \right\}
\]

In the remainder of this scenario we restrict the analysis to values of \(v\) and \(w\) that satisfy thresholds from (7) and (8). Demand aggregation for an available product \(jk\) follows (5) and firm profits from possible product configurations of \(j \in \{I, E\}, k = L\) are given by

\[
\pi_I = p_{IL} \cdot Q_{IL} + w \cdot Q_{EL}
\]

\[
\pi_E = (p_{EL} - w) \cdot Q_{EL}.
\]

As the incumbent operates the legacy network, the entrant has to buy access from the incumbent via the access fee \(w\). Therefore, the incumbent receives, in addition to the profits from its own product, also payments dependent on the entrant’s demand for its basic product. Hence, the entrant can only partially extract the rents from its own basic access product while some portion of it is expropriated by the incumbent via \(w\).

**Private equilibrium**

In the private equilibrium firms’ prices for all \(n = 2\) product configurations and the wholesale access fee \(w\) are the strategic decision variables. We solve for the solution to the private maximization problems via backward induction. Based on product \(jk’\)’s demand function from (5) and firms’ profit functions in (9) and (10), we differentiate with respect to prices and produce the set of first order conditions (FOCs) which is given below.

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10This stands in contrast to the following Scenario II in which (4) equals zero due to \(N = 4, n = 4\) in this case.

11Please note that these do not depend on which unserved spoke was chosen as first preference since both are unavailable and offer the identical utility of zero.
\begin{align*}
  p_{IL} &= \frac{2v}{5} + \frac{t}{10} + \frac{w}{10} + \frac{p_{EL}}{10}, \\
  p_{EL} &= \frac{2v}{5} + \frac{t}{10} + \frac{w}{2} + \frac{p_{IL}}{10}. 
\end{align*}

Naturally for price competition under horizontal differentiation, a rival’s product price and more pronounced consumer preferences via \( t \) are increasing a product \( jk \)'s own price. Prices are independent of \( \delta \) since new technology products are non existent. The wholesale price \( w \) has a positive effect on both product prices but a stronger one on \( p_{EL} \). Given that \( w \) decreases the entrant’s profit margin from its own legacy demand, see (10), she increases its price more strongly in order to dampen the loss in its margin. These effects persist if one solves this system of FOCs and develops price reaction functions (12) and demands (13) in dependence on \( w \).

\begin{align*}
  p_{IL}(w) &= \frac{4v}{9} + \frac{t}{9} + \frac{5w}{33}, \\
  p_{EL}(w) &= \frac{4v}{9} + \frac{t}{9} + \frac{17w}{33}. \\
  q_{IL}(w) &= \frac{5}{108} + \frac{5w}{27t} - \frac{2w}{99t}, \\
  q_{EL}(w) &= \frac{5}{108} + \frac{5v}{27t} - \frac{20w}{99t}.
\end{align*}

Given that both firms only offer one product, trade-offs that may result from different pricing incentives between the products are not present in this Scenario I. Demands for both products depend negatively on \( w \). We summarize as our first result the following.

**Result 1**: If both the entrant and the incumbent offer only a legacy access product (Scenario I), the wholesale access fee positively (negatively) affects prices (demands). Trade-offs resulting from different pricing incentives between multiple products of a single firm are not present. The access fee \( w \) acts only as rent extraction device to the advantage of the incumbent.

Entering (12) and (13) into (10) and (9) one can formulate the incumbent’s and entrant’s profit functions in dependence on the access fee \( w \) to be the following:

\begin{align*}
  \pi_I(w) &= \frac{5(t + 4v)^2}{972t} - \frac{670w^2}{3267t} + \frac{91(t + 4v)w}{1782t}, \\
  \pi_E(w) &= \begin{cases} 
  \frac{5(t+4v)^2}{972t} + \frac{320w^2}{3267t} - \frac{80(t+4v)w}{1782t} & \text{if } w \leq \frac{11v}{12} + \frac{11t}{48}, \\
  0 & \text{if otherwise.}
  \end{cases}
\end{align*}
The incumbent’s profits exhibit non-monotonicity in \( w \) as she faces a trade-off. Intuitively, a higher value of \( w \) results in a higher margin that is transferred to the incumbent at the intensive margin. However, \( w \) is paid in dependence of the entrant’s demand for legacy connections which is negatively affected by \( w \), see (13). This negative effect at the extensive margin ensures concavity of \( \pi_I(w) \) such that the solution to the optimization of the incumbent’s profits indeed characterizes a maximum. This private optimal \( w \) can be derived to be

\[
    w^* = \frac{1001v}{2010} + \frac{1001t}{8040} > 0
\]

(15)

It can be shown that \( w^* \) also satisfies previously established restrictions from (8) such that \( w \leq w^* \leq \overline{w} \), with \( w = 0 \). Figure 4 displays both firms’ profit functions and \( w^* \) for the eligible parameter range.

Figure 4: SI - Private optimal access fee \( w \) (\( t = 1, v = 1.75 \))

Welfare Optimum

While market prices are exclusive strategic firm decisions, determining the access fee may be the responsibility of a regulatory agency which acts as a social planner. Ruling out any dual mandate in the objective of this social planner, we assume that the relevant objective is to maximize social welfare as the aggregate of consumer and producer surplus. While the former is determined by summing both expressions in (14), the latter is calculated as the sum of all consumer utilities from each pairwise spoke comparison and is formalized by (6). Since prices are nevertheless privately chosen, we use solutions from (21) and (23) to determine the access fee dependent welfare as follows.

\[
    W(w) = \frac{260tv + 520v^2 - 89t^2}{1944t} - \frac{175w^2}{3267t} - \frac{2(t + 4v)w}{81t}
\]

(16)
One can see already from (16) that welfare depends negatively on $w$ and, hence, is a decreasing function for eligible parameter values of $v$ and $t$. This is an intuitive result as a higher wholesale price consequences lower higher prices, lower demands for NGN products and also an increased consumer drop-out and purchasing of the zero utility numeraire. Differentiation of (16) with respect to $w$ produces

$$w_{SP} = -\frac{484v}{525} - \frac{121t}{525} < 0$$

which is strictly negative. Consequently, a welfare maximizing social planner would set the lowest possible value of $w_{SP}^* = 0$. Figure 5 displays the relation of welfare and firms’ profits. We summarize and formulate this as our next result.

**Result 2:** In Scenario I, the socially optimal access fee $w_{SP}^*$ is a corner solution at zero. Hence, the private choice of the wholesale fee $w^*$ always exceeds that of a social planner who tries to minimize higher product prices and limit consumer NGN drop-out with a lower fee.

Figure 5: SI - Socially optimal access fee $w \ (t = 1, v = 1.75)$
Scenario II - Full deployment

This scenario resembles competition in a situation in which the higher quality network infrastructure has been fully deployed and each of the feasible product configurations is available such that \( n = 4 \). Hence, this resembles the end-point of a evolutionary deployment path and an operation of both infrastructures, legacy and new technology, in parallel. This may already be the case today in situations in which multiple network operators have found a specific area viable to each roll out a congruent higher quality network (fiber). Such a scenario is applicable to European highly densely populated areas like metropolitan cities or areas in which co-investment models have successfully been applied.

Consumer preferences and available products (firm locations) in this scenario are displayed in the right panel of Figure 1. In contrast to Scenario I, all product configurations are available, that is, \( j \in \{ I, E \} \), \( k \in \{ H, L \} \). A consumer located on spoke \( l_{jk} \) has always her first product preference of \( jk \) available to her, as well as her second preference of \( j'k' \). Given that the base valuation \( v \) is large enough, no consumer considers dropping out of the NGN market and buying the zero-utility numeraire. Consequently, the market is fully covered and product demands add up to unity in equilibrium.\(^{12}\)

In the following notations referring to a product \( jk \), we iterate over the full set of available varieties, that is, \( j \in \{ I, E \} \), \( k \in \{ H, L \} \). In this way, firms’ profits are determined in the following as:

\[
\pi_I = p_{IH} \cdot Q_{IH} + p_{IL} \cdot Q_{IL} + w \cdot Q_{EL} \quad (18)
\]

\[
\pi_E = p_{EH} \cdot Q_{EH} + (p_{EL} - w) \cdot Q_{EL} \quad (19)
\]

Again the incumbent operates the legacy network to which the entrant buys access at the fee of \( w \). Therefore, the incumbent receives, in addition to its own profit streams, also payments dependent on the entrant’s demand for its legacy product. Hence, the entrant can only fully extract the rents from its own higher quality product since profits from copper services are partially expropriated by the incumbent via \( w \). This difference in economic value between the entrant’s two demand segments gives rise to the key pricing incentives in this scenario.

Private equilibrium

In the private equilibrium firms’ prices for all \( n = 4 \) product configurations and the wholesale access fee \( w \) are strategic decision variables. Again we solve for the solution to the private maximization problems via backward induction.

Using a product \( jk \)’s demand function from (5) and firms’ profit functions in (18) and (19) for differentiation with respect to prices, one arrives at the set of first order conditions

\(^{12}\)Since no spoke is unserved, the demand segment of those consumers formalized in (4) equals zero. From this also follows that Scenario I does not exhibit restrictions on \( w \) comparable to (8) from Scenario I.
(FOCs) which is given below.

\[
\begin{align*}
    p_{IH} &= \frac{t}{2} + \frac{\delta}{3} + \frac{w}{6} + \frac{p_{EH} + p_{EL}}{6} + \frac{p_{IL}}{3} \\
    p_{IL} &= \frac{t}{2} - \frac{\delta}{3} + \frac{w}{6} + \frac{p_{EH} + p_{EL}}{6} + \frac{p_{IH}}{3} \\
    p_{EH} &= \frac{t}{2} + \frac{\delta}{3} - \frac{w}{6} + \frac{p_{IH} + p_{IL}}{6} + \frac{p_{EL}}{3} \\
    p_{EL} &= \frac{t}{2} - \frac{\delta}{3} + \frac{w}{2} + \frac{p_{IH} + p_{IL}}{6} + \frac{p_{EH}}{3} \\
\end{align*}
\]

It becomes apparent that each rival’s product prices are increasing product \( jk' \)'s price in a symmetric fashion. However, intra-brand competition or own-cannibalization towards the respective other product of the same supplier is a concern to the effect that price increases in \( p_{jk'} \) are discounted at twice the rate when deciding on \( p_{jk} \).

Intuitively, \( \delta \) increases the price of the higher quality products and affects prices of legacy products negatively. In this way, firms capture some of consumers’ higher valuation for those products also with higher prices. Additionally, the more pronounced consumer preferences are, that is, the larger \( t \), the higher are product prices which is the standard result of horizontal differentiation.

Comparative statics of prices with respect to the access fee \( w \) are more nuanced and will be discussed on the basis on the solutions to the system of FOCs in (20). Naturally, these depend only on the remaining decision variable of \( w \) and can also be characterized as reaction functions in this regard. These are:

\[
\begin{align*}
    p_{IH}(w) &= \frac{3t}{2} + \frac{\delta}{4} + \frac{w}{2} \\
    p_{IL}(w) &= \frac{3t}{2} - \frac{\delta}{4} + \frac{w}{2} \\
    p_{EH}(w) &= \frac{3t}{2} + \frac{\delta}{4} + \frac{w}{4} \\
    p_{EL}(w) &= \frac{3t}{2} - \frac{\delta}{4} + \frac{3w}{4} \\
\end{align*}
\]

The comparative effect of the access fee \( w \) in product prices is twofold. First, it acts as a positive pricing premium for all four product configurations which is passed-through to consumers, although it is actually only paid by the entrant. Second, the pass-through of the wholesale access fee is symmetric for the incumbent but asymmetric for the entrant to the effect that \( \frac{\partial p_{EL}}{\partial w} > \frac{\partial p_{EH}}{\partial w} \). Intuitively, the entrant tries to promote its higher quality product with a lower price compared to its own legacy one. In this way the entrant can economize on wholesale costs and simultaneously boost the demand for its own high quality product of which he can extract rents fully. This demand steering effect through \( w \) by the entrant is the driving force of equilibrium solutions and characterizes our next main result.
**Result 3:** If the entrant offers both a basic and a high quality access product (Scenarios I & IV), the wholesale access fee $w$ gives rise to a demand steering effect by the entrant as she passes on the wholesale access costs towards consumers in an asymmetric manner. Precisely, it holds that $\frac{\partial p_{EL}}{\partial w} > \frac{\partial p_{EH}}{\partial w}$. In doing so, demand for the entrant’s higher quality product is promoted at the expense of the entrant’s own, less lucrative, legacy product.

Using prices from (21) one can derive the resulting product demands in dependence of $w$ to be

$$
q_{IH}(w) = \frac{1}{4} + \frac{\delta}{12t}, \\
q_{IL}(w) = \frac{1}{4} - \frac{\delta}{12t}, \\
q_{EH}(w) = \frac{1}{4} + \frac{\delta + w}{12t}, \\
q_{EL}(w) = \frac{1}{4} - \frac{\delta + w}{12t}.
$$

Two aspects are noteworthy here. First, higher quality products benefit from an increased demand based on the technology’s quality advantage $\delta$ relative to consumer preferences $t$, whereas legacy products’ demand suffers. This manifests in the term of $\frac{\delta}{12t}$ which is either added or subtracted from $\frac{1}{4}$ as the symmetrical demand split between all four product configurations.

Second, demand for the incumbent’s products is independent from $w$, while demands for the entrant’s products are not. Recall from prices in (21) that $w$ serves as a pricing premium which is passed onto consumers for all products. While the incumbent lifts prices symmetrically by $\frac{w}{2}$, the entrant distributes these premiums asymmetrically to promote its higher quality product. However, on average, all product prices increase by $\frac{w}{2}$ such that no competitive effects in resulting demand shifts materialize with respect to this level. Hence, only the asymmetric pass-through of the entrant persists and is reflected in a bonus (malus) to its high (low) quality product’s demand. The independence of the incumbent’s legacy demand of $w$ further implies that there is a certain share of consumers that will always stick to the lower value product of $jk = IL$. Therefore, a total adoption of the new infrastructure will not be achievable by changes in $w$, only by changes in $\delta$ and $t$. We formulate these observations as our fourth main result.

**Result 4:** In Scenario II, demand for the incumbent’s products is independent of $w$. Hence, demand shifts within those consumer segments can only be induced by the relation between the quality advantage of new technology’s products $\delta$ and the intensity of consumer preferences $t$. Demand effects through higher levels of $w$ materialize in the form of migration from the entrant’s own legacy product towards her high quality product as a result of asymmetric pass-through.
The incumbent’s demand function for its legacy product is also the origin of our initial restriction on the incremental quality advantage in the form of \( \delta \leq 3t \). This exactly satisfies non-negativity of the second expression in (22) as it solely depends on the relation of quality advantage and intensity of consumer preferences and is unaffected by \( w \). However, the other non-negativity restriction that originates from the last expression in (22) is \( q_{EL} \geq 0 \), which requires \( w \leq 3t - \delta \). For values of \( w \) which exceed this threshold, we have to restrict \( q_{EL} \) to be zero to ensure non-negativity. Based on this, one can formulate the incumbent’s and entrant’s profit functions in dependence on the access fee \( w \) to be the following:

\[
\begin{align*}
\pi_I(w) &= \frac{\delta^2 + 18t^2}{24t} - \frac{w^2}{12t} - \frac{w(\delta - 6t)}{12t}, \\
\pi_E(w) &= \frac{\delta^2 + 9\delta t + 18t^2}{48t} + \frac{w^2}{48t} + \frac{w(2\delta + 9t)}{48t} + \frac{6t - \delta - w}{4} \cdot \begin{cases} 
\frac{1}{4} - \frac{\delta + w}{12t} & \text{if } w \leq 3t - \delta \\
0 & \text{if otherwise}
\end{cases}.
\end{align*}
\]

It can be shown, that the entrant’s profits are strictly increasing in \( w \), while the incumbent’s are not. The access fee positively influences own product prices as well as the margin the incumbent gets from the entrant’s legacy product demand. However, the size of the entrant’s legacy demand from which she can extract \( w \) is negatively affected. This constitutes a classic trade-off between profits at the extensive and intensive margin. Differentiating the incumbent’s profits from (23) with respect to \( w \), provides the private optimal value of the access fee which maximizes incumbent’s profits \( w^* \) as follows.

\[
w^* = 3t - \frac{\delta}{2} \quad \text{(24)}
\]

However, this unbounded private optimal access fee \( w^* \) always exceeds the value of \( 3t - \delta \) and, thus, implies that demand for the entrant’s legacy product is zero and has been completely migrated to other products which necessarily aligns with the entrant’s pricing incentives. This is displayed in Figure 6 below for values of \( t = 1, \delta = 1 \) which satisfy the ex-ante parameter condition of \( \delta \leq 3t \).

Intuitively, \( w^* \) increases in \( t \) since consumer preferences are more pronounced and demand for legacy connections is less price sensitive. Contrarily, a larger level of \( \delta \) has a negative impact as it implies a stronger quality advantage of the new technology which results in consumers substituting away from legacy products more willingly. We summarize and conclude the section on private optimization with our next main result.

**Result 5:** In Scenario II, private optimal wholesale fee \( w^* \) implies zero demand for the entrant’s legacy product \( (q_{EL}(w^*) = 0) \). A strong pass-through of access costs by the entrant leads to a full migration away from its own legacy product in equilibrium.
Welfare Optimum

If a welfare maximizing social planner or regulator is in charge of choosing \( w \), she will do so such as to maximize the following function:

\[
W(w) = \frac{\delta^2}{8t} + \frac{\delta(w + 12t)}{24t} - \frac{12t^2 - 48v \cdot t + w^2}{48t}
\]  \hspace{1cm} (25)

Differentiation with respect to \( w \) and solving the resulting FOC produces the socially optimal access fee of

\[
w_{SP}^* = \delta .
\] \hspace{1cm} (26)

Hence, the socially optimal level of the access fee \( w_{SP}^* \) corresponds to the incremental quality advantage of the new technology \( \delta \). Consequently, the larger the utility benefits from the higher quality access products are, the higher the social choice of \( w \) which intensifies the migration from legacy to new technology between the entrant’s products (see from (22)). This dynamic stands in contrast to the privately chosen access fee \( w^* \) which depends negatively on \( \delta \) (see from (24)). Furthermore, the socially optimal access fee \( w_{SP}^* \) is smaller than the private choice \( w^* \) if

\[
\delta \leq 2t
\] \hspace{1cm} (27)

is satisfied. Compared to our ex-ante restriction on the parameter space of \( \delta \leq 3t \), it becomes clear that this threshold is well within the eligible parameter range. Hence, Scenario I exhibits cases of over- and underprovision in \( w \) compared to the socially optimal solution. Figure 7 displays welfare implications for previous parameters of \( t = 1, \delta = 1 \) and, hence, the case of overprovision in \( w \).
The intuition behind the social planner choice is the following. If $\delta$ as the quality advantage of new technology products is relatively large, that is, the inequality in (27) is less likely to hold, then the social planner chooses a higher access fee $w$ in order to enhance legacy to new technology migration between the entrant’s products. In other words, it is socially desirable that as much consumers as possible benefit from the added utility component of $\delta$ compared to the cost of choosing a product that is farther away from one’s original preference $t$. However, if $\delta$ is relatively small compared to $t$, the gains outweigh the incurred transportation costs only to a lesser degree and the socially optimal access fee $w_{SP}^*$ shrinks in relation to $w^*$.\footnote{Please note that full migration away from $jk = EL$ has already occurred before the inequality in (27) is no longer satisfied. The motivation for a social planner to increase $w_{SP}^*$ even above $w^*$ is then only substantial utility gains through a high $\delta$ compared to losses through $t$.} We summarize and formulate insights from the welfare section as the next result.

**Result 6:** In Scenario II, the socially optimal access fee equals the incremental quality advantage of the new technology $w_{SP}^* = \delta$. The privately chosen access fee $w^*$ exceeds the socially desirable level if $\delta < 2t$. Hence, the larger (smaller) the quality advantage of the new technology $\delta$ in relation to the intensity of consumer preferences $t$, the higher (lower) is $w_{SP}^*$ in relation to $w^*$ to foster (deter from) the legacy to new technology migration between the entrant’s products.
Scenario III - Incumbent first deployment

In Scenario III competition between the incumbent and the entrant is over both legacy products and one new technology offer by the incumbent. This resembles a market situation in which the incumbent moved first to roll-out new infrastructure and beat the entrant to be the first to offer new technology access. Situations in which an incumbent can leverage its legacy technology also to roll-out new technology faster are best represented by this Scenario. Available product configurations are \( n = 3; j \in \{I, E\}; k = \{L, H\} \) and there is the potential for numeraire drop-out for consumers located on \( l_{EH} \). See Figure 8.

Figure 8: S1 - Preference space, available products and numeraire drop-out

The calculation of demands for a product \( jk \) follows the same path as in the previous Scenario analyses. From restrictions on consumer locations in (4), one can, again, derive upper and lower bounds on \( v \) and \( w \) which are the following.

\[
\begin{align*}
\nu &= \nu_{IH} = \frac{128t - 47\delta + 24w}{56}; \\
\nu &= \max \left\{ \nu_{EL} = \frac{16t - 3\delta + 26w}{30}, \nu_{IL} = \frac{32 - 3 + 24w}{56} \right\} \\
\omega &= \omega_{EL} = \frac{30v - 16t + 3\delta}{26}; \\
\omega &= \max \left\{ \omega_{EL} = \frac{30v - 62t + 3\delta}{26}, \omega_{IH} = \frac{56v - 128t + 47\delta}{24}, 0 \right\}
\end{align*}
\] (28)

(29)

In the remainder of this scenario we restrict the analysis to values of \( v \) and \( w \) that satisfy thresholds from (28) and (29). Firm profits from possible product configurations are given by

\[
\begin{align*}
\pi_I &= p_{IH} \cdot Q_{IH} + p_{IL} \cdot Q_{IL} + w \cdot Q_{EL} \\
\pi_E &= (p_{EL} - w) \cdot Q_{EL}
\end{align*}
\] (30) (31)
Private equilibrium

Given that the derivation of the private and social equilibrium solutions follow the same procedures as in the two previous Scenarios, these sections will be brief and only highlight the important findings. Hence, we directly present optimal prices and quantities in dependence of $w$ that result as solutions to the system of FOCs stemming from differentiation profit functions of (30) and (31) with respect to prices.

$$p_{IL}(w) = \frac{9v}{23} + \frac{9t}{23} + \frac{6w}{23} - \frac{\delta}{92},$$

$$p_{EL}(w) = \frac{8v}{23} + \frac{8t}{23} + \frac{13w}{23} - \frac{6\delta}{92},$$

$$p_{IH}(w) = \frac{9v}{23} + \frac{9t}{23} + \frac{6w}{23} + 45\delta.$$

$$q_{IL}(w) = \frac{9}{92} + \frac{9v}{92t} - \frac{5w}{276t} - \frac{49\delta}{1104t},$$

$$q_{EL}(w) = \frac{8}{69} + \frac{8v}{69t} - \frac{40w}{276t} - \frac{24\delta}{1104t},$$

$$q_{IH}(w) = \frac{9}{92} + \frac{9v}{92t} - \frac{5w}{276t} + \frac{181\delta}{1104t}.$$

Results from previous scenarios carry over into this situation in which $\delta$ positively affects price and demands for $jk = IH$, while it has the adverse effect on prices and demands for both legacy products. Similarly, the access fee $w$ acts as a price premium for all product prices and the incumbent again chooses to distribute this symmetrically. Given that the entrant offers only a legacy product, strategic cross promotion via asymmetric pass-through of $w$ is not possible and her choice is mainly motivated by dampening the loss in its profit margin. Firm profits are be derived to be\(^{14}\)

$$\pi_I(w) = T_I(v, t, \delta) - \frac{245w^2}{1587t} + \frac{5(194t + 194v + \delta)w}{6348t},$$

$$\pi_E(w) = \begin{cases} 
T_E(v, t, \delta) + \frac{100w^2}{1587t} - \frac{40(16t+16v-3\delta)w}{6348t} & \text{if } w \leq \frac{16v+16t-3\delta}{20}, \\
0 & \text{if otherwise},
\end{cases}$$

and differentiation of $\pi_I$ with respect to $w$ produces

$$w^* = \frac{97v}{196} + \frac{97t}{196} + \frac{\delta}{392} > 0.$$  

The incumbent will always choose an optimal access fee $w^*$ which is positive. Further-

\(^{14}\)Please note that $T_I(v, t, \delta)$ serves as a strictly positive placeholder term which depends positively on each of its three arguments $v, t$ and $\delta$. This term is omitted for the sake of an easier representation of firms’ profits while not limiting the interpretation with respect to the choice of $w$. 

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more, it can be shown that it always satisfies lower and upper bounds defined in (29) such that \( w \leq w^* \leq \bar{w} \). The left panel in Figure 9 displays both firms’ profit functions and \( w^* \) for the eligible parameter range.

**Figure 9: SIII - Private & social optimal access fee \( w \) \((t = 1, \delta = 1, v = 1.75)\)**

**Welfare Optimum**

If a social planner or regulator decides on \( w \), she will do so by maximizing social welfare of

\[
W(w) = T_W(v, t, \delta) - \frac{145w^2}{3174t} - \frac{5(82t + 82v - \delta)w}{6348t}
\] (36)

with the socially optimal choice of

\[
w_{SP} = -\frac{82v}{116} - \frac{82t}{116} + \frac{\delta}{116} < 0
\] (37)

which is strictly negative and always below the privately chosen value, such that \( w_{SP} < 0 < w^* \). This is also displayed in the right panel of Figure 9. Again, welfare is a strictly decreasing function in \( w \) since there is no upside and no potential to beneficially steer demand from legacy to new technology products due to the symmetric pass-through of \( w \) by the incumbent. Higher levels of the access fee, therefore, only lead to an increased drop-out to the numeraire, comparable to effects in Scenario I. Given our eligible parameter range defined by (29) the social planner will choose the smallest possible value as corner solution of \( w_{SP}^* = \bar{w} \). We summarize the findings of Scenario III as our next main result.

**Result 7:** If the Incumbent deploys the new technology access first in parallel to existing legacy products (Scenario III), product prices increase and demands decrease in \( w \). The price effect of \( w \) is symmetric between the incumbent’s two products and provides no potential for inducing legacy-to-
new migration via choice of \( w \) (for the social planner). Consequently, the socially optimal choice of \( w_{SP}^* \) is the lowest eligible value that minimizes losses in consumer utility and therefore consumers dropping-out from the NGN market.

**Scenario IV - Entrant first deployment**

In Scenario IV competition between the incumbent and the entrant is over both legacy products and one new technology offer by the entrant. This resembles a market situation in which the entrant moved first to roll-out new infrastructure and beat the incumbent to be the first to offer new technology access. Real world situations which are best represented through this scenario are for instance local operators that started primarily as a wholesale customer of the incumbent via LLU, but opted according to the ladder-of-investment theory, to invest in own new access infrastructure. Available product configurations are \( n = 3; j \in \{I, E\}; k = \{L, H\} \) and there is the potential for numeraire drop-out for consumers located on \( l_{IH} \), See Figure 10.

Figure 10: S1 - Preference space, available products and numeraire drop-out

The calculation of demands for a product \( jk \) follows the same path as in the previous Scenario analyses. Relevant restrictions on consumer locations in (4), are, again, derived as upper and lower bounds on \( v \) and \( w \) which are the following.

\[
\begin{align*}
\bar{v} &= v_{EH} = \frac{128t - 47\delta + 3w}{56}; \quad v = v_{EL} = \frac{36t - \delta + 49w}{56} \\
\bar{w} &= w_{EL} = \frac{56v - 36t + \delta}{49}; \quad \underline{w} = \max \left\{ \frac{56v - 128t + \delta}{49}, 0 \right\} 
\end{align*}
\]
In the remainder of this scenario we restrict the analysis to values of $v$ and $w$ that satisfy thresholds from (38) and (39). Firm profits from possible product configurations are given by

$$\pi_I = p_{IL} \cdot Q_{IL} + w \cdot Q_{EL}$$  \hspace{1cm} (40)$$

$$\pi_E = p_{EH} \cdot Q_{EH} + (p_{EL} - w) \cdot Q_{EL}.$$  \hspace{1cm} (41)

Private equilibrium

Similar to the previous section, we will be brief on the derivation of the private and social equilibrium solutions and directly present optimal prices and quantities in dependence of $w$ that result as solutions to the system of FOCs stemming from differentiating profit functions of (40) and (41) with respect to prices.

$$p_{IL}(w) = \frac{8v}{23} + \frac{8t}{23} + \frac{18w}{92} - \frac{6\delta}{92},$$  \hspace{1cm} (42)$$

$$p_{EL}(w) = \frac{9v}{23} + \frac{9t}{23} + \frac{49w}{92} - \frac{\delta}{92},$$

$$p_{EH}(w) = \frac{9v}{23} + \frac{9t}{23} + \frac{3w}{92} + \frac{45\delta}{92}.$$  \hspace{1cm} (43)

$$q_{IL}(w) = \frac{8v}{69} + \frac{8v}{69} t - \frac{5w}{276t} - \frac{24\delta}{1104t},$$

$$q_{EL}(w) = \frac{9v}{92} + \frac{9v}{92} t - \frac{175w}{1104t} - \frac{49\delta}{1104t},$$

$$q_{EH}(w) = \frac{9v}{92} + \frac{9v}{92} t + \frac{55w}{1104t} + \frac{181\delta}{1104t}.$$  \hspace{1cm} (43)

Optimal prices and quantities are similar to those of previous Scenarios with respect to their dependence on $t$ and $\delta$. The most striking result is the pass-through of the wholesale prices $w$ onto prices and, ultimately, quantities. Since the entrant now offers both a legacy and a new technology product he chooses two price asymmetrically in $w$ and increases the prices of its legacy product and cuts it for the new technology. Precisely, it holds that $\frac{\partial p_{EL}}{\partial w} > \frac{\partial p_{EH}}{\partial w}$ which gives rise to the same cross-subsidization and demand steering effect by the incumbent already known from Scenario II. Nevertheless, the wholesale price acts as a price premium for all products. Firm profits are be derived to be

$$\pi_I(w) = T_I(v, t, \delta) - \frac{4115w^2}{25392t} + \frac{5(580t + 580v - 241\delta)w}{25392t},$$  \hspace{1cm} (44)$$

$$\pi_E(w) = \begin{cases} T_E(v, t, \delta) + \frac{3845w^2}{50784t} - \frac{5(432v + 432t - 2656)w}{25392t} \text{ if } w \leq \frac{108v + 108t - 496}{175}, \\ T''_E(v, t, \delta) + \frac{55w^2}{33856t} + \frac{5(384v + 384t + 5039)w}{16926t} \text{ if otherwise}. \end{cases}$$
and differentiation of $\pi_I$ with respect to $w$ produces

$$w^* = \frac{290v}{823} + \frac{290t}{823} - \frac{241\delta}{1646} \quad (45)$$

The incumbent will always choose an optimal access fee $w^*$ which strictly lower than that of Scenario III from (35) in which it is the incumbent who offers the only new technology product. This is especially the case, since $\delta$ affects $w^*$ now negatively in (45). Intuitively, if the quality advantage of the new technology $\delta$ is large, the the optimal access fee $w$ is rather low in order to limit the entrant’s price promotion and loss of demand for the incumbent. Furthermore, it can be shown that it always satisfies lower and upper bounds defined in (29) such that $w < w^* < \overline{w}$, with $w = 0$. The left panel in Figure 11 displays both firms’ profit functions and $w^*$ for the eligible parameter range.

Figure 11: SIV - Private and socially optimal access fees $w$ ($t = 1, \delta = 1, v = 1.25$)

**Welfare Optimum**

If a social planner or regulator decides on $w$, she will do so by maximizing social welfare of

$$W(w) = T_W(v, t, \delta) - \frac{4385w^2}{101568t} - \frac{5(496t + 496v - 277\delta)w}{50784t} \quad (46)$$

with the socially optimal choice of

$$w_{SP} = -\frac{496v}{877} - \frac{496t}{877} + \frac{277\delta}{877} < 0 \quad (47)$$

which is strictly negative given the eligible parameter ranges in this scenario is simultaneously smaller than the privately chosen value, such that $w_{SP} < 0 < w^*$. This is also displayed in the right panel of Figure 11. Although, welfare is still a strictly decreasing function in $w$ the socially optimal choice in this scenario exceeds that of the other hybrid Scenario III. Precisely, $\delta$ has a stronger positive effect on $w_{SP}$ in (47) compared to
With this the social planner takes into account that she can influence the migration towards $jk = EH$ positively with a higher $w$. Intuitively, this is more beneficial, if the utility advantage of the new technology $\delta$ is large. Nevertheless, higher levels of the access fee, lead to higher prices as an counteracting force and still imply increased drop-out to the numeraire. Hence, the social planner will choose the smallest possible value that satisfies (39) as corner solution of $w_{SP} = 0$. We summarize the findings of Scenario IV as our next main result.

**Result 8:** If the entrant deploys the new technology access first in parallel to existing legacy products (Scenario IV), product prices increase and demands decrease in $w$. The price effect of $w$ is asymmetric between the entrant’s two products and induces demand migration towards its new technology product at the expense of its legacy one, such that $\frac{\partial q_{EH}}{\partial w} > \frac{\partial q_{EL}}{\partial w}$ holds. The social planner respects this pricing structure but still chooses $w_{SP}$ as the lowest eligible to minimize losses in consumer utility and therefore consumers dropping-out from the NGN market.
4 Maximizing migration to the new technology

In the scenario analyses of the previous Section 3 we assumed a welfare maximizing social planner or regulator. Although maximizing welfare should be the ultimate goal of economic institutions, it is also reasonable that this mandate may be shifted towards other objectives for certain periods in time, e.g., maximizing consumer demand migration towards the new technology. Among the potential regulatory toolkit to achieve this goal is the choice of the access fee \( w \) which is already established as part of LLU obligations for the legacy infrastructure.

For differentiation with previously determined levels of the privately chosen fee \( w^* \), the welfare maximizing fee \( w^*_{SP} \), we introduce the concept of a “migration optimal” social access fee \( w^*_H \) which maximizes aggregate demand for products \( jk \), with \( k = H \), in a given scenario. In cases that exhibit multiple values of \( w \) (candidates) which maximize aggregate demand of the new technology, \( w^*_H \) selects the candidate that produces the highest welfare. Naturally, a departure from the socially optimal access fee, that is, \( w^*_H \neq w^*_{SP} \), implies welfare losses. However, in this analysis these are of secondary importance since the prime objective is to maximize migration to the new technology. We define the aggregate demand for the new technology as follows

\[
Q_H = \sum q_{jH}, j \in \{I, E\}. \tag{48}
\]

Since the new technology is not yet available in Scenario I, we restrict the subsequent analysis to the remaining Scenarios. In the ultimate state of the deployment process represented by Scenario II, aggregate demands of the new technology include consumers from both the incumbent and the entrant. Recall from Results 4 to 6 that \( Q_H \) increases in \( w \) up to the point at which all consumers migrated away from the entrant’s legacy product while incumbent’s product demands are independent of \( w \). Starting from \( w^*_{SP} \) the access fee could be increased to \( w^*_H = 3t - \delta \) which maximizes migration while minimizing welfare losses. Furthermore, \( w^*_H \) ensures the highest possible migration but at a lower social cost compared to the private solution of \( w^* \), see Figure 12.

This additional demand for the high technology product can be calculated to be \( \Delta Q_H = Q_H(w^*_H) - Q_H(w^*_{SP}) \), with \( \Delta Q^H_I = \frac{1}{4} - \frac{\delta}{6t} \) for Scenario II. Given our chosen parameters in this scenario, a regulator could induce an additional \( \Delta Q^H_I(t = 1, \delta = 1) = 1/12 \) of the entire consumer mass to purchase a product based on the new technology. In relative terms this would imply an increase of 11.11% compared to the level resulting from choosing \( w^*_{SP} \).

We showed that the regulator can use the access fee in an ultimate state of a deployment process. Subsequently, we turn to intermediate Scenarios III & IV which reflect the path towards that final state. In these Scenarios the demand for the new technology \( Q_H \) is either exclusively served by the incumbent (Scenario III) or by the entrant (Scenario IV).
If the incumbent deploys new infrastructure first, demands for all products depend negatively on $w$ such that the socially optimal access fee is identical to the migration optimal one, that is, $w_{SP}^* = w_H^*$. The symmetric pass-through of $w$ on incumbent’s prices gives no room for the regulator to improve upon the welfare maximizing solution with the aim to foster migration (see left panel of Figure 13). Hence, $\Delta Q_{III}^H = 0$ holds.

If the entrant deploys the new infrastructure first (Scenario IV), it follows from Result 8, that the asymmetric pass-through of $w$ by the entrant leads to $\frac{\partial Q_H}{\partial w} > 0$. While a welfare maximizing social planner also takes into account consumer utilities, the migration maximizing regulator does so only as a second priority. Hence $w_H^* = w$ not only exceeds the socially optimal solution $w_{SP}^*$ but also the private one $w^*$ (right panel of Figure 13). With this choice, the regulator is able to increase demand for the new technology.
by $\Delta Q_{IV}^H = \frac{1045 v}{122278} - \frac{55(1386+79t)}{515676}$. Given our eligible parameter space in this scenario this would equate to $\Delta Q_{IV}^H (t = 1, \delta = 1, v = 1.25) = 0.08325$ of the entire consumer base. In relation to the socially optimal fee this would imply an increase of 24.75%. We summarize the findings of this section as our next main result.

**Result 9**: The access fee is an effective tool to enhance demand migration from legacy to new technology in Scenarios in which the entrant offers a new technology product (II & IV). Given our concept of the migration optimal access fee $w^*_H$, a regulator can increase take-up of the new technology compared to the welfare maximizing level $w^*_SP$. The foundation of this effect lies in the entrant’s asymmetric pass-through of the access fee $w$ into his product prices (Result 3). Given our eligible parameters, increases in new technology take-up range from $\Delta Q_{II}^H = \frac{1}{12}$ to $Q_{IV}^H = 0.08325$ of the consumer mass. Naturally, choosing $w^*_H$ implies losses in social welfare.
5 The optimal deployment path

The differentiation between scenarios allows us not only to analyze the specific states of a deployment process in isolation but we can also conjecture on the possible deployment paths towards full deployment. Starting from Scenario I, either the incumbent or the entrant can move first in deployment of the new technology. As previous results show, the competitive environment and also the potential to steer demand migration are significantly different between the intermediate Scenarios III & IV. Hence, the route deployment takes matters!

We use (36), (46) and define $\Delta W^{IV-III} = W^{IV} - W^{III}$ as the welfare advantage in a state where the entrant moves first to deploy (Scenario IV) compared to a first move by the incumbent (Scenario III). It can be shown that $\Delta W^{IV-III} > 0$ unambiguously holds and that it is beneficial from a welfare perspective if the entrant leads the way in deploying the new infrastructure (see blue path in Figure 14). Furthermore, we have shown in Section 4 that the path through such a Scenario IV also preserves the potential to positively influence also the take-up of the new technology through $w$. Conclusively, the path of $I \rightarrow IV \rightarrow II$ quickly enables the entrant’s crucial pricing incentives and to achieve this path, deployment of the entrant should be encouraged.

Figure 14: Welfare optimal deployment path

Originating from Scenario I the incentive of a firm $j$ to move first with the deployment of the new infrastructure can be characterized by the gains it would receive through such a move. For the incumbent this is given by $\pi^{I \rightarrow III}_I = \pi^{III}_I - \pi^{I}_I$ stemming from (14) and (34). Analogously, for the entrant this is characterized by comparing profits of Scenarios I and IV such that $\pi^{I \rightarrow IV}_E = \pi^{IV}_E - \pi^{I}_E$.

From the perspective of a social planner or regulator, this gives room for another objective when deciding on the access fee $w$ apart from maximizing social welfare. In Scenario I, the regulator could choose $w$ in such a way, that it maximizes the entrant’s incentives to deploy the new infrastructure first, while it minimizes that of the incumbent. Therefore, the regulator can influence which deployment path is likely to materialize.

Both relevant profit differences are displayed in Figure 15 and it can be shown that both are increasing functions in $w$, that is, $\frac{\partial \pi^{I \rightarrow III}_I}{\partial w} > 0$ and $\frac{\partial \pi^{I \rightarrow IV}_E}{\partial w} > 0$. Hence, an increase in $w$ in Scenario I would imply that the gains through a first move by the incumbent and the entrant increase. However, for eligible parameters the absolute value of these deployment
gains differ. Furthermore, gains of an first moving entrant are concave in \( w \left( \frac{\partial^2 \pi_{E \rightarrow IV}}{\partial w^2} < 0 \right) \) while they are convex for a first moving incumbent \( \left( \frac{\partial^2 \pi_{I \rightarrow III}}{\partial w^2} > 0 \right) \). Figure 15 highlights that this gives rise to a range of \( w \) for which an entrant’s first move gains outweigh the incumbent’s. We denote the difference in these deployment gains as

\[
F_E = \pi_{E \rightarrow IV} - \pi_{I \rightarrow III}
\]  

(49)

to which we refer in the following as the entrant’s first move incentives. Intuitively, for positive (negative) values of \( F_E \), gains of deploying first exceed (are smaller than) the incumbent’s. Hence, it is reasonable to assume that if \( F_E > 0 \) it is more likely that a deployment path via Scenario IV will materialize.

Figure 15: First moving incentives of the entrant in \( w \) (\( t = 1, \delta = 1, v = 1.75 \))

If a regulator in a Scenario I type situation decides on \( w \), she can do so in order to maximize \( F_E \) and increase the likelihood that the preferred deployment path will arise. The access fee which maximizes \( F_E \) can be calculated to be

\[
w_{F_E}^* = \frac{122312 v - 118912 t}{122235} + \frac{99 \delta}{281} > w^{SP}.
\]  

(50)

One can show that \( w_{F_E}^* \) exceeds not only the welfare optimizing solution of the starting Scenario I \( w_{SP}^* = 0 \) but also the level that is privately chosen by the incumbent \( w^{SP} \) (see Figure 15 in yellow). Hence, a regulator who wants to promote the beneficial deployment path of an entrant’s first move, will increase the wholesale fee substantially. From Figure 4 we see that this significantly reduces profits of the entrant in Scenario I, which, in turn, increases the potential profit gains the entrant can receive if she would depart from that state and deploy new infrastructure first.\(^{15}\) Conclusively, we formulate our last main

\(^{15}\) Naturally, we abstract in this analysis from deployment costs that would arise when switching between scenarios.
Result 10: The optimal deployment path involves the entrant to deploy the new technology first (Scenario IV). Precisely, $I \rightarrow IV \rightarrow II$ results in a higher social welfare and provides additional steering options of high technology take-up in the intermediate term though choice of $w$. To maximize an entrant’s incentives to deploy first relative to the incumbent, the regulator can choose $w = w_{FE}^* > w^*$. This reduces an entrant’s profits in the starting Scenario I but, consequently, increases the gains through an early deployment of the infrastructure.
6 Conclusion

This paper analyzes technology competition between two firms in different scenarios, which vary in the availability of products. Consumers’ preferences, however, have already been formed for all possible product varieties. The main interpretation of this model framework can be seen in an evolutionary deployment process of a new network infrastructure. While access products based on the legacy network infrastructure are still being offered in parallel, deployment of the new technology can take different paths. Either firm, that is, the incumbent or the entrant, can be the first to deploy the new infrastructure and to offer a product as alternative to the legacy access. The strategic variable of interest in our analysis is the wholesale access fee \( w \) that is paid per-unit by the entrant to the incumbent in relation to the entrant’s demand for her legacy product. Model features of different scenarios (available product varieties), multi-product firms, and the transfer payment \( (w) \) between firms all expand the Spokes model of horizontal differentiation between multiple products by Chen and Riordan (2007). Our analysis produces the following results.

The access fee for legacy products generally increases equilibrium prices of all products. Although the fee is only paid by the entrant, both firms pass-on a proportion of this to consumers prices. The exception to this is the pricing behavior of the entrant if she offers a new technology products in parallel to her legacy one. If this is the case (Scenarios II & IV), the wholesale fee increases the legacy product’s price over proportionally negative, to the benefit of a lower price for the new technology product. This cross-promotion and asymmetric pricing leads to a steering of demand away from the less lucrative (legacy) product to the new product whose rents are fully captured by the entrant.

The positive pass-through of the wholesale fee to product prices implies that consumers pay more for their purchases, potentially purchase a product that is less congruent to their inherent product preferences, or even drop-out of the market and opt for a zero utility numeraire. All of which exert a negative effect on social welfare and, hence, the socially optimal access fee is smaller than the privately optimal fee chosen by the incumbent.

If welfare is of a lesser concern and a regulator is more interested in promoting take-up of the new technology, the access fee is an effective tool to do so. The asymmetric pass-through of the fee to entrant’s product prices can be utilized by a regulator to strategically promote the migration away from legacy products towards the new technology. Naturally, a prerequisite for this is that the entrant has deployed the new infrastructure and offers both technologies in parallel (Scenarios II & IV). In doing so, the regulator will choose a higher access fee compared to the socially optimal level and accepts a lower welfare to the benefit of a stronger legacy-to-new migration. Noteworthy in this context is, that at the end of the deployment path when both firms operate new infrastructure (Scenario II), consumers with preference of the entrant’s legacy product migrate fully to the new technology while the incumbent’s legacy customers cannot not be influenced by the
access fee and form a resilient consumer group.

The path towards a full supply of product varieties (infrastructure deployment) in Scenario II implies either firm to move first in their supply (deployment) decision. We find that a first move of the entrant, that is, Scenario IV is not only from a welfare optimal but it also offers a regulator the option to influence take-up of the new technology via the access fee. Hence, increasing the entrant’s incentive to deploy first may be of interest. Starting from Scenario I, a regulator can strategically increase the access fee to reduce an entrant’s present profits, which simultaneously increases the gains through an early deployment of the infrastructure and, hence, increasing the likelihood that the optimal path will materialize.

Our results are relevant for political and regulatory decision-making especially, but not limited to, the context of already existing access fees as part of LLU. While access fees have been necessary to open up markets and enable competition on consumer level, regulators may be increasingly concerned with competition between different infrastructures and lacking take-up. We show that existing access fees for legacy infrastructures give rise to new pricing incentives in connection with a new technology that makes access fees a promising tool to enhance welfare, new technology take-up and influencing the sequence of deployment moves.
References


