Copper to Fibre Migration: Regulated Access Fees Incentivising Migration

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Abstract

To shed more light on consumer-sided demand migration, we adapt Chen & Riordan’s (2007) Spokes Model of spatial competition to a duopolistic-multi-product firm setting in which both firms simultaneously offer fibre and copper products comparable to Brito & Tselekounis (2017). Our model will be designed as a 2x2-product Spokes Model where two operators, an Incumbent and an Entrant, offer each a fibre and a copper based end customer internet product, with the Entrant paying an access fee for the latter one. Deriving operators’ profits given demand shifts induced by asymmetric pricing strategies, we find that both operators experience trade-offs in the wholesale access fees, with the trade-off of the Incumbent being more binding as he has the higher interest in keeping demand in the copper network high. We characterise the relation of fibre take-up and welfare by finding out that fibre take-up and welfare both increase simultaneously in the copper wholesale access fee up to a critical threshold. Beyond this threshold, additional take-up will be paid by loss of total welfare.
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1 Introduction

Investments into fibre infrastructure have increased in the past decades throughout the European Union. According to a recent OECD statistic, cable remains the dominant fixed-line broadband technology in OECD countries, accounting for 34% of subscriptions. However, with a rise of 5.6% in 2020, fibre is steadily replacing DSL subscriptions, which have dropped by 10% over the past two years.\(^1\) This roll-out has been accompanied by discussions on how to foster the investment in and take-up of this future-proof infrastructure further.

Many initial considerations have focused on creating a regulatory environment which favours investments into fibre infrastructure, the foundation of actual fibre take-up. Besides this focus on roll-out, European and national broadband goals are increasingly shifting the focus from mere availability of fibre to active usage of this technology. As stated in the 2018 established European Electronic Communications Code “The regulatory framework should, in addition to the existing three primary objectives of promoting competition, the internal market and end-user interests, pursue an additional connectivity objective, articulated in terms of outcomes: widespread access to and take-up of very high capacity networks for all citizens of the Union and Union businesses […]” (European Parliament & European Council, 2018). Moreover, authorities explicitly state take-up as part of their broadband goals also on a national level. A growing number of thus far ten European member states have included take-up goals as part their national broadband plans\(^2\). On a European level, it becomes evident that the mere existence of a fibre-based network is a necessary yet not sufficient condition for their active usage. While the availability of FTTH/B networks has reached a relevant coverage, subscription numbers still fall far behind (see Figure 1).

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Therefore, current political and regulatory frameworks evidently are successful in achieving the main goal of incentivising fibre investments, at least to some extent, but lack solutions on how to improve the actual adoption of fibre where the technology is already available. This issue has also been the subject of a public consultation in the context of the French LLU decision in 2020. The French National Regulatory Authority (NRA) discussed the evolution of LLU prices in the context of migrating access lines from copper to fibre. As Neumann et al. (2020) state it in the given context: „This […] means that even where fibre is available the majority of users still satisfy their communications demand by using Oranges copper network and not the fibre networks. […] A gap between demand and supply of fibre is not in line with the targets of the French fibre infrastructure policy and it is not in line with the economically efficient outcome“.

This paper aims at contributing to the literature on demand sided fibre adoption and copper-fibre migration. Whereas existing literature has thus far analysed migration primarily under the caption of investments, one can observe that an assumed fibre take-up as a result of investments into the infrastructure cannot be taken for granted.

In an effort to generate comparable results to such literature, we set out to analyse the issue of fibre take-up by the same means. As our focus is on migration, we consider fibre infrastructure as prerequisite in our analysis, as we aim to shed light on the question of how political decision makers may use their means of copper access pricing to foster fibre adoption and which welfare effects have to be considered in this decision process.

The remainder of this study is structured as follows: Chapter 2 will summarise the relevant scientific literature in the context of our study. In chapter 3 we will introduce our basic model specifications and derive the results of our baseline scenario and compare these findings in comparative static analyses before discussing the implications for fibre adoption and welfare. We conclude this chapter by deriving implications for regulators in
chapter 4 and policy-makers. Finally, we summarise the results of our analysis’ and give an outlook on potential further research on this topic in chapter 5.

2 Relevant Literature

The analysis of market conditions in the transition from copper to fibre network has been subject to various researches mostly relying on theoretical modelling. In the context of fibre take-up and adoption, previous researchers have often relied on the Hotelling Model (Hotelling, 1929) and related models.

Existing research mostly focusses on either the implications of a copper access charge or fibre access charge on investment into an FTTH network. The literature identified a trade-off between dynamic and static efficiency when assessing fibre access charges and their relation with investment incentives. Static efficiency is reached in a case of low access prices increasing competition yet undermining incentives for the network operators to invest in an FTTH network or expanding existing footprint therefore decreasing dynamic efficiencies (Flacher & Jannequin, 2012). Flacher & Jannequin focus their analysis on the fibre segment of the market and confirm the decrease of investment incentives in the case where the fibre network is accessible at regulated fees. Also they show that the total welfare is higher in the unregulated scenario. In order to counter-steer the effect of decreased investment incentives, they suggest to combine access regulation with geographic coverage commitments. The authors assume the revenues on the copper segment of the businesses to be zero and assume the fibre network and price-setting to be dependent on investments into the fibre network. We will abstract from such assumption setting as we will consider firstly, the copper business segment as historically relevant for operators’ revenues and therefore pricing decisions, secondly we will consider investments to be irrelevant for pricing as we intend to focus on customer rather than operators’ migration incentives.

Bourreau, Cambini & Dogan (2012) have identified three counter-steering effects of the “Replacement Effect”, “Wholesale Revenue Effect” and “Business Migration Effect” when analysing the impact of copper access charges on investment incentives. The authors apply a model containing investment costs and investment modelling in order to investigate conflicting effects of investment incentives caused by access regulation of two coexisting infrastructures. They find that an access fee on the old network will increase the Incumbent’s profit only to a certain limit as the wholesale buyers will at this limit either not continue purchasing access or invest into an FTTH-network. The latter will depend on the degree of NGA (Next Genearion Access)-coverage of the incumbent which the Entrant will only contest if the Incumbent’s NGA-footprint is relatively small. Also they show that in this case of a small Incumbent’s footprint, the Entrant will roll-out an NGA-network and act just like a monopolist would even though to some extent the Entrant will have duplicated the Incumbent’s NGA-network. The extent of the Incumbent’s footprint the Entrant will be willing to conquest is depending on the access
fee. A higher access fee incentivises the alternative network operator (ANO) to invest and compete with the Incumbent to a greater extent. The authors define this effect of ANOs being triggered to invest into infrastructure as access charges increase as “Replacement Effect”. The authors find two more effects that occur when assessing investment incentives of Incumbents and ANOs as simultaneously to the replacement effect, a high copper access charge hampers investments by the Incumbent as his investments would be followed by investments of the ANOs which again would lead to reduced wholesale revenues. Bourreau, Cambini & Dogan label this effect as the “Wholesale Revenue Effect”. They identify a third investment effect, which affects customer-sided migration. They label this effect as the “Business Migration Effect”. At a high copper access charge, retail prices are high as well which incentivises end customers to migrate towards the retail product of higher quality as price difference is relatively small. While Bourreau, Cambini & Dogan approach is closest to our research intention of analysing the impact of a copper access fee on migration, we shall focus on the consumer-sided effects of their analysis applying a different model approach free of investments. As investments of one operator may exclude roll-out of the other one, we shall not consider investments as we intend not to limit consumer migration streams by availability.

Jeanjean and Liang (2011) employ a Hotelling Model of vertical and horizontal competition in order to investigate the impact of wholesale copper on fibre investments. Vertical competition is captured in additional value which is being attributed to the fibre product by end users. The authors investigate competition of two operators in a situation where the copper-operating Incumbent, an alternative network operator (ANO) or both may invest into fibre infrastructure. They find that under the assumption of perfect competition, i.e. $t = 0$, the market share of copper is smaller if the Incumbent invests into fibre and the ANO does not. Also they find, an increase in copper access charges would increase incentives for one of two operators to invest into fibre but decrease incentives for both operators to invest simultaneously. Further, under the assumption that copper and fibre customers are segmented strongly by their individual preference for copper or fibre, i.e. consumers regard copper and fibre as very different, the higher the copper access charge is, the greater is their incentive to migrate to fibre. In that case, an increase of copper access charges increases the ANOs incentives to invest. Given that the authors make their analysis based on Hotelling-modelling, they are limited to comparing two products at a time. We shall shed further light on consumer implications as we shall enable consumers to choose between four offers simultaneously by applying the Spokes Model for four products of copper and fibre offered each by an Incumbent and an Entrant.

Tselekounis, Orfanou & Varoutas (2014) claim to be the first authors to have been analysing competition between fibre and copper between two firms which both may offer copper and fibre based services simultaneously. Jeanjean & Liang had assumed that as soon as one operator had invested into fibre, service provisioning on copper was stopped. Tselekounis; Orfanou & Varoutas, by means of the Hotelling Model, seek to
determine the copper and fibre wholesale access charges which incentivise fibre investment without distorting competition. They find that the incentives for the Incumbent to invest into fibre infrastructure are optimal when access to the copper network is priced at provisioning costs and the fibre access is priced at the level which maximises the Incumbent’s profit. This finding follows the assumption that maximising the Incumbents profits will lead to him investing into fibre. They find that the fibre access charge maximising the Incumbent’s profits at a given copper wholesale charge is reached when the margins from providing FTTH services perfectly outweigh the effect of customers preferring the copper based product amid high end customer prices for the fibre based product. Just like Jeanjean & Liang, Tselekounis, Orfanou & Varoutas rely on the Hotelling Model for their analysis of investment implications and competition of copper and/or fibre access charges. Again we shall open up the analysis to a setting where copper and fibre products of two operators are in simultaneous competition with one another by applying the Spokes Model as we are more interested in consumer migration than operators’ profits. For the latter such limitation may be less critical.

While the existing literature mostly aims at investigating circumstances that maximise FTTH investment incentives while finding optimum trade-offs for static efficiency and shedding light on the copper switch-off process from the point of view of network operators, we will focus the demand shifts for coexisting networks under copper regulation. Therefore, while the literature identified different implications and trade-offs of copper and fibre wholesale fees for investment incentives, we will concentrate solely on consumer behaviour and how it may be induced by a copper wholesale fee.

![Figure 2 Embedment of the Current Research Target in the Timeline](image)

Figure 2 Embedment of the Current Research Target in the Timeline

Therefore, this paper adds to the existing literature by investigating another stage on the timeline where NRAs already successfully maximised incentives for fibre investments, but actual take-up is not matching speed. One could argue that where Incumbents are the investor of fibre infrastructure, there may be an intrinsic incentive for that operator to switch off the legacy network in order to save operation costs. Yet Tenbrock, Knips, & Wernick (2020) find that this incentive diminishes as over time operational expenditures of the operation of both networks will converge. Therefore, the
longer networks coexist, the lower may become the intrinsic motivation for an Incumbent to migrate his consumers, which as a consequence, may have to be induced by policy makers.

We will base our analysis on the Spokes Model introduced by (Chen & Riordan, 2007) that was also the basis for the analysis of coexistence of copper and fibre infrastructures that Brito & Tselekounis had applied in 2017. The authors apply a Spokes Model with 4 spokes with two operators offering each a product based on a fibre and on a copper network. While in their model the access to the copper network is free of charge, access to the fibre network requires paying a wholesale access fee by one operator the other. The authors apply their model focussing on the effect of multi-product competition on the operator’s profits under access regulation to the fibre network. They find that an increase in the fibre wholesale access fee increases the Entrant’s profit as the copper product becomes more attractive to consumers at high prices for the fibre product while the Incumbent’s profit was U-shaped in the fibre access fee. Also they find conditions where regulator and Incumbent would apply the same fibre access fee. We will take this model as the basis of our considerations yet switching the logic of access around: The copper network will be subject to access regulation while two operators will be assumed to have rolled out a fibre network, which as a consequence will be free of access charges and investments will be considered sunk. We assume such setting to be more applicable to the current situation of roll-out and to be suitable for our purpose of analysing the influence of the copper wholesale access fee on customer migration. Furthermore, Brito & Tselekounis a priori split up the consumer base into technology-oriented and firm-oriented subpopulations. We will dispense with this assumption as we are trying to analyse costumer migration and therefore intend to not apply constraints of product-migration.

3 Model Description

Our model-setup relies on a modified version of Chen & Riordan 2007 established Spokes Model. In the Spokes Model, every product specification is located on one of \( N \) spokes. If only two product specifications at two firms are considered, the Spokes Model collapses to the standard Hotelling Model\(^3\), which only captures the price and localisation decision of two competitors.

\(^3\) Introduced by (Hotelling, 1929)
In our setting we aim at capturing a scenario where

1) the copper network is not switched off immediately as soon as fibre has been rolled out and

2) the adoption of existing fibre infrastructure is influenced by the (potentially regulated) access fee for the legacy copper network due to duplication of infrastructure.

We focus on demand shifts in a scenario of coexisting networks and for this purpose consider a region where investment in terms of footprint have been completed and therefore do not consider implications for investments of networks. In our view such assumption is most suitable for analysing demand-side aspects of copper to fibre migration and combining the analysis of demand and operator incentives of migration within the same model would lose sight on at least one of both aspects.

Like in the model set-up of the paper most related to ours of Brito & Tselekounis (2017), we consider two competing network operators – one Incumbent and one Entrant – that compete for the consumer base where each firm internalises two spokes, i.e. two firms each offer two products, one lower quality product (copper based) and one higher quality product (fibre based). Likewise we assume the firms and their products to be located at the extremes of the spokes.

Unlike the model of Brito & Tselekounis (2017), where access to the copper network is possible at zero costs for both firms and the Entrant may offer a fibre based product in return for paying an access charge to the Incumbent, we consider a model where the copper based product is accessible to the Entrant at an access charge and the fibre network had been rolled out by the Incumbent and the Entrant as well to the identical footprint and is therefore free of access charges. Consequently, two firms offer two products each and investment costs are considered to be sunk and therefore not relevant for pricing decisions of both firms. Also marginal costs are assumed to be zero. Both assumptions are line with recent literature researching effects of co-existing telecommunications infrastructures like in the model of Brito & Tselekounis from 2017.
Figure 3  Model Design in the Spokes Model

Firms compete for a mass of consumers on $N = 4$ spokes normalised to 1. Consumers have the following utility function

$$U_{ij} = v + \delta_{ij} - tx - p_{ij}$$

(1)

with $v$ being the base value assigned by a consumer to internet access, irrespectively of the underlying technology, $\delta_{ij}$ being the incremental value a consumer assigns to receiving a product from firm $i$ with $i \in (I, E)$ based on the technology $j$ with $j \in (L, H)$. Consumers incur linear transport costs $t$ for the distance $x$ from where is located to the product location of product $ij$ and additionally the price for the respective product $ij$. Every consumer will purchase exactly one unit and the base utility $v$ is assumed to be high enough for every consumer to purchase exactly one unit of the offered products. In the context of internet accesses it is reasonable to assume that each consumer will only purchase exactly one unit.

Brito & Tselekounis (2017) divide the total population of consumers into two sub-populations, each caring only for two options at a time. Either consumers prefer a specific firm and decide between the lower and higher quality product of this brand, or they prefer a specific technology and then decide between the firms that offer products based on this technology. By this assumption, which is a restriction of the original Spokes Model by Chen & Riordan (2007), consumers that are indifferent between the opposing technology and firm are excluded from the consumer base, e.g. there would
no consumers that are indifferent between the lower quality product of the Entrant and the higher quality product of the Incumbent. We do not follow this assumption as it a priori limits the potential streams of consumer migration that we are keen to analyse but instead return to the original specification of the Spokes Model. Therefore, the position of the indifferent consumer between two products (in our case, the end points of two spokes) is determined by:

$$x_{ij/ml} = \frac{1}{2} + \frac{(p_{ml}-p_{ij}) + (\delta_{ij}-\delta_{ml})}{2t}.$$  

(2)

In a setting where both technologies are offered by both companies (i.e. no spoke is unoccupied), the demand for a product $ij$ is given by:

$$q_{ij} = \frac{2}{N} \sum_{m \neq i, j \in \{1, 2\}} \left\{ \min \left\{ \frac{1}{2} + \frac{(p_{ml}-p_{ij}) + (\delta_{ij}-\delta_{ml})}{2t}, 1 \right\}, 0 \right\}$$  

(3)

with $N = 4$ in our context.

Firms optimise their profits over prices. However, the profit function of the Incumbent and Entrant differ, since the Entrant is (to some extent) access seeker. As the Incumbent operates the legacy copper network, the Entrant has to buy access from the Incumbent (access charge $w$). Therefore, the Incumbent benefits from transfer payments due to customers of the Entrant that do not opt for an access product based on fibre technology. The Incumbent makes profit in his own fibre as well as in his copper business segment and also receives revenue from his wholesale copper business as he gives access to the Entrant to this network for the wholesale access fee of $w$. The Entrant at the same time yields profit from his own fibre and copper segment while he has wholesale costs for the latter one:

$$\pi_I = p_{IH}q_{IH} + p_{IL}q_{IL} + wq_{EL}$$  

(4)

$$\pi_E = p_{EH}q_{EH} + (p_{EL} - w)q_{EL}$$  

(5)

We will assume a social planner in some form of political decision maker to be aware of consumers’ and firms’ behaviour and as being responsible for determining on whether or not the level of wholesale access fee should be subject to regulation. His criterion for this assessment will be the outcome of welfare under private and social optimum that will later be complemented by the criterion of the level of fibre adoption in the market reached under both possibilities. Welfare will be defined as consumer and producer surplus reached within the demand and supply of internet products on the model spokes.

### 3.1 Scenario 1: Competition in the copper-only case

Before solving the Spokes Model in the scenario of full availability of fibre and copper infrastructures outlined above, we investigate price setting and regulated wholesale
access fees in the scenario of competition in a legacy copper network under competition. Without fibre technology being available, the Spokes Model collapses to the standard Hotelling Model with \( N = 2 \).

\[ (IL) \text{ spoke } IL \]

\[ (EL) \text{ spoke } EL \]

Figure 4 Model Design in the Pure Copper Scenario

Without loss of generality we assume that \( \delta_i I = 0 \), meaning that consumers have no differentiated valuation for the copper products offered by both operators in terms of perceived quality. This assumption is justified by the fact that the Entrant has to buy access to the legacy copper network of the Incumbent and therefore the quality of the underlying network is identical for customers of both companies.\(^4\)

In line with the standard Hotelling results, the demand functions for both companies are:

\[ q_{IL} = \hat{x}_{IL} = \frac{1}{2} + \frac{(p_{EL} - p_{IL})}{2t} \] \hspace{1cm} (6)

and

\[ q_{EL} = 1 - \hat{x}_{IL} = \frac{1}{2} + \frac{(p_{IL} - p_{EL})}{2t} \] \hspace{1cm} (7)

The profit functions of both firms are again influenced by the level of the access fee \( w \):

\[ \pi_I = p_{IL} q_{IL} + w q_{EL} \] \hspace{1cm} (8)

\[ \pi_E = (p_{EL} - w) q_{EL} \] \hspace{1cm} (9)

The resulting equilibrium prices for both firms are

\[ p_{IL} = w + t, \] \hspace{1cm} (10)

\(^4\) In this analysis we abstract from any form of strategic quality degradation or sabotage by the incumbent.
at which both firms serve exactly half of the population and firms receive the following profits:

\[ \pi_I = w + \frac{t}{2} \]  \hspace{1cm} (11)

\[ \pi_E = \frac{t}{2} \]  \hspace{1cm} (12)

The profit of the Entrant is independent of the wholesale access fee for copper which implies that he is able to pass on the costs of purchasing the wholesale service to the end customers. Therefore, aggregated consumer surplus yields (see Annex I)

\[ CS = v - \frac{5t}{4} - w \]  \hspace{1cm} (13)

which shows that the access fee is in fact borne by the consumers to the full extent.

The corresponding total welfare is given by:

\[ WF = v - \frac{t}{4} \]  \hspace{1cm} (14)

**Insight 1:**

Total welfare in the case of pure copper competition is independent of the level of the access fee \( w \), as it is being directly passed on to consumers by both network operators.

In this scenario of sole copper competition, the Incumbent in the market equilibrium would see no intrinsic motivation of not setting an arbitrarily high access fee to his network as it is firstly a transfer payment from the Entrant to the Incumbent and secondly it raises the end customer prices.

A social planner in this case would see sufficient motivation to determine the access fee for the copper infrastructure of the Incumbent as opposed to leaving the setting the fee to the Incumbent. As the welfare is neutral with respect to \( w \). The social planner will set an access fee that will conserve market conditions, namely the competitive situation of an Incumbent and an access seeker, that is able to compete with the Incumbent at retail level, and a continuum of consumers that have demand for the goods. The condition for market participation of consumers and producers is reached at:

\[ w_{\text{max}} = v - t \]  \hspace{1cm} (15)

At \( w_{\text{max}} = v - t \) the utility of consumers intending to purchase at the Entrant is zero and therefore he is indifferent between purchasing and not purchasing any good, given equilibrium prices and quantities. Therefore, if \( w \) exceeds this level, this would imply that demand for the product of the Entrant would be zero. This upper bound for the...
access fee is somewhat heuristic as by model definition, \( v \) is always guaranteeing that consumers will purchase an offered good by one of the both operators.\(^5\)

### 3.2 Scenario 2: Coexistence of Copper and Fibre

In the second scenario, the higher quality product becomes available to the consumer base as two network operators have found a certain area viable to each roll out a congruent fibre network. Such scenario today may be applicable to European highly densely populated areas like metropolitan cities or to areas where co-investment models have successfully been applied. The scenario may as well be interpreted as futuristic scenario where fibre has been sufficiently supplied by at least one operator but the copper network has not yet been switched off.

This setting is described by the Spokes Model with \( n = 2 \) firms at \( N = 4 \) spokes which all are occupied by existing products. For the sake of simplicity we assume, that \( \delta_{II} = \delta \) and, just like in 3.1, \( \delta_{IL} = 0 \) which implies that consumers make no differentiation within the same level of productivity of the product between both firms but are able to acknowledge the technological advantage of the higher quality product which is fibre based. This assumption is in line with the assumption set of Brito & Tselekounis (2017).

#### 3.2.1 Market Equilibrium Outcome

The social planner in the pure copper scenario was not able to determine a wholesale access fee for the copper network that optimises welfare but could only determine a upper bound for this parameter. This also implies that under new market conditions, the social planner will have to reassess the market in order to determine a defined wholesale access fee. Therefore, given equilibrium quantities, both firms will optimise retail prices for the lower and higher value products. The social planner, in full awareness over equilibrium prices and quantities will then determine the copper wholesale access fee \( w \) by backward induction and secondly will determine outcomes if the Incumbent was left to set the wholesale access fee. On this basis he will decide on whether or not he should intervene in the market. His criterion for this assessment will be the outcome of welfare under private and social optimum that will later be complemented by the criterion of the level of fibre adoption in the market reached under both possibilities.

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\(^5\) In reality access fees are based on complex cost-based models to determine a reasonable price that covers the costs of the incumbent of providing access to its network. However, even if there is an absolute upper bound to the valuation of consumers (\( \overline{v} \)), our assumption could be easily adapted to the upper bound of consumer’s valuations exceeding this cost-based threshold. In other words, consumers are assumed to be willing to cover the factual costs of providing internet access to their home.
3.2.2 Equilibrium prices

Firms optimise their prices given their profit functions by inserting quantities under (3) into the profit functions (4) and (5) and solving the system of first order conditions. By backwards induction, we can derive the prices both firms will set simultaneously in anticipation of all other prices.

The Incumbent’s reaction on anticipated prices for the higher quality product is:

\[ p_{IH} = \frac{t}{2} + \frac{\delta}{3} + \frac{w}{6} + \frac{p_{EH}}{6} + \frac{p_{EL}}{6} + \frac{p_{IH}}{3} \]  

(16)

and for the lower quality product:

\[ p_{IL} = \frac{t}{2} - \frac{\delta}{3} + \frac{w}{6} + \frac{p_{EH}}{6} + \frac{p_{EL}}{6} + \frac{p_{IH}}{3} \]  

(17)

An increase in the respectively other price of \( p_{ij} \) gives exactly double the leverage as an increase of the prices of the competitor’s spokes would. Also, both prices increase in \( w \), therefore \( w \) can be interpreted as a means of lifting the total pricing level. What is more, intuitively, \( \delta \) increases the price the Incumbent is able to demand for its high quality product while to the same extent it decreases the price of its lower quality product.

As the Incumbent is able to internalise price effects on both of his spokes, prices can be expressed as solely dependent on the Entrant’s pricing:

\[ p_{IH} = \frac{3t}{4} + \frac{\delta}{4} + \frac{w}{4} + \frac{p_{EH}}{4} + \frac{p_{EL}}{4} \]  

(18)

and

\[ p_{IL} = \frac{3t}{4} - \frac{\delta}{4} + \frac{w}{4} + \frac{p_{EH}}{4} + \frac{p_{EL}}{4} \]  

(19)

Simultaneously, the Entrant forms his prices in anticipation of the Incumbent’s price setting. His depending prices are for the higher quality product

\[ p_{EH} = \frac{t}{2} + \frac{\delta}{3} - \frac{w}{6} + \frac{p_{IH}}{6} + \frac{p_{IL}}{6} + \frac{p_{EH}}{3} \]  

(20)

and consequently for the lower quality product

\[ p_{EL} = \frac{t}{2} - \frac{\delta}{3} + \frac{w}{2} + \frac{p_{IH}}{6} + \frac{p_{IL}}{6} + \frac{p_{EH}}{3} \]  

(21)

With regard to the respectively other own prices of \( p_{Ej} \), interpretation is corresponding to the one of the Incumbent’s prices as price increases in the own spokes can be partly internalised. Yet the interpretation of the role of \( w \) is worth explaining further. If \( w \) is large, then the Entrant sees incentive of lowering the price of its higher valued product in order to shift demand onto its fibre network as this is the network he faces no
wholesale costs for. Simultaneously, the price for the lower value product is increasing in $w$.

The Entrants' price functions that solely depend on the prices he is unable to steer yield:

$$p_{EI} = \frac{3t}{4} + \frac{\delta}{4} + \frac{p_{II}}{4} + \frac{p_{IL}}{4}, \quad \text{and}$$

$$p_{EL} = \frac{3t}{4} - \frac{\delta}{4} + \frac{w}{2} + \frac{p_{II}}{4} + \frac{p_{IL}}{4}$$

(23)

As equations (22) and (23) show, for the internalised prices of the Entrant, only the lower value product has a (positive) dependence on $w$.

Solving the reaction prices of (20) – (23) yields the following equilibrium prices:

$$p_{II} = \frac{3t}{2} + \frac{\delta}{2} + \frac{w}{2}$$

(24)

$$p_{IL} = \frac{3t}{2} - \frac{\delta}{2} + \frac{w}{2}$$

(25)

$$p_{EH} = \frac{3t}{2} + \frac{\delta}{4} + \frac{w}{4}$$

(26)

$$p_{EL} = \frac{3t}{2} - \frac{\delta}{4} + \frac{3w}{4}$$

(27)

This setup of equilibrium prices shows that, just like in scenario 1 of sole copper competition, both firms pass on the wholesale access fee to the end customer while in the case of coexistence of copper and fibre, they have each two products at hand to forward this effect. While both firms pass on the wholesale fee, the Incumbent does so symmetrically via both products while the Entrant does so asymmetrically (75% on copper, 25% on fibre). By this pricing strategy, the Entrant follows two objectives: Firstly, he intends to motivate demand shifts from his copper product in order to reduce wholesale costs and secondly, he tries to win customers for his fibre product from the three other spokes. As a positive side effect, the Entrant is able to cross-subsidise wholesale costs for copper via his fibre price.

**Insight 2:**

The Entrant's copper prices face the highest increase in the copper wholesale access fee, his fibre prices the lowest. Both Incumbent's prices have the same slope in $w$. 

3.2.3 Equilibrium quantities

Given the known equilibrium prices, the position of the indifferent consumers gives us the demand shifts that incur in an increase in \( w \).\(^6\) We find that, as the Incumbents’ prices increase symmetrically in \( w \), consumers who have as first and second preference Incumbent’s products will not be able to make themselves better off by switching to their second preferred product. What is more, we can see that, to the exact same extent as the Incumbent wins customers from the Entrant’s copper product (where price increase is the highest in \( w \)), he loses them to the Entrant’s fibre product (where price increase is the lowest in \( w \)). We can therefore conclude that the Entrant’s pricing strategy proves to be successful for his objectives as the Entrant’s copper customers overall face the highest incentives to switch to their second preferred product and his fibre clients the lowest. In other words, no customer with the Entrant’s copper product being his second preference will see incentives to switch to this second preferred product. At this stage we can see that a certain amount of consumers will not be migratable to the fibre infrastructure by \( w \).

Given equilibrium prices, equilibrium quantities in dependence of the wholesale access fee as the result of the described migration effects and consumer behaviour in prices yield:

\[
q_{IH} = \frac{1}{4} + \frac{\delta}{12t} \quad (28)
\]

\[
q_{IL} = \frac{1}{4} - \frac{\delta}{12t} \quad (29)
\]

\[
q_{EH} = \frac{1}{4} + \frac{\delta + w}{12t} \quad (30)
\]

\[
q_{EL} = \frac{1}{4} - \frac{\delta + w}{12t} \quad (31)
\]

Non-negativity of quantity is granted as the following four conditions hold:

1. \( 0 \leq w \leq 3t - \delta \)
2. \( 0 \leq \delta \leq 3t \)
3. \( \frac{d}{3} \leq t \leq \frac{w}{3} + \frac{\delta}{3} \)
4. \( 0 < t \)

At the upper ends of the boundaries for \( w \) and \( t \), the two higher value products share the market equally while the quantities of the two lower value products is exactly 0.

Quantity functions show that, as a result of demand shifts in prices described in the beginning of this chapter, only the quantities of the Entrant change in \( w \) therefore all shifts between copper and fibre induced by the copper access fee would incur in demand shifts within the consumer base of the Entrant, while quantities of the

\(^6\) The locations of the respective indifferent consumers can be found in ANNEX II
Incumbent remain constant under changes in $w$. The independency of $p_{IL}$ of $w$ implies that there is a certain share of consumers that will always stick to the lower value product of the Incumbent and therefore a total adoption of the fibre infrastructure will not be achievable by changes in $w$. Further we observe that the incremental valuation of fibre over copper $\delta$, leads to an increase in fibre demand to the same extent as it leads to a decrease in copper demand while the opposite holds for the behaviour of quantities with regard to transactional costs $t$.

**Insight 3:**

Not all quantities will increase in $w$, therefore a certain amount of consumers will not be able to be motivated to switch to fibre by steering the copper wholesale fee.

The total fibre adoption shall further be defined as:

$$q_H = q_{IH} + q_{EH} \quad (32)$$

### 3.2.4 Consumer surplus and producer surplus

At known prices and locations of the indifferent consumers of $x_{ij/m_l}$, the consumer surplus can be derived as follows:

$$CS = \sum_{m \neq i, i \in (1,2), j \in (1,2)} \frac{1}{N} \int_0^{x_{ij/m_l}} v + \delta_{ij} - tx - p_{ij} \, dx \quad (33)$$

Total consumer surplus then yields:

$$CS = \frac{w^2}{48t} - \frac{7\tau - 2(w + \delta)}{4} + \frac{\delta(w + \delta)}{24t} + v \quad (34)$$

The consumer surplus for each of the four spokes can be seen in ANNEX III.

The negative term in $w$ describes that consumers are in general worse off in an increase in the copper access fee, as it consequently leads to an overall increase of the retail price level. The quadratic term in consumer surplus in $w$ is a result of the ability of consumers to switch to their second preferred product as the price of the first preferred product exceeds the price level of the second preferred product. This demand shift weakens the negative effect of the overall price increase.

The producer surplus is the sum of profits of both operators. Both operators internalise two spokes while the Incumbent is in addition partly able to internalise the $EL$ spoke by gaining $w$ as a wholesale revenue.

Profits of (4) and (5) in dependence of $w$ can be written as

$$\pi_I = \left( \frac{3t}{2} + \frac{\delta}{4} + \frac{w}{2} \right) \times \left( \frac{1}{4} + \frac{\delta}{12t} \right) + \left( \frac{3t}{2} - \frac{\delta}{4} + \frac{w}{2} \right) \times \left( \frac{1}{4} - \frac{\delta}{12t} \right) + w \times \left( \frac{1}{4} - \frac{\delta + w}{12t} \right) \quad (35)$$
And

\[
\pi_E = \left( \frac{3t}{2} + \frac{\delta}{4} + \frac{w}{4} \right) \ast \left( \frac{1}{4} + \frac{\delta + w}{12t} \right) + \left( \frac{3t}{2} - \frac{\delta}{4} + \frac{3w}{4} \right) \ast \left( \frac{1}{4} + \frac{\delta - w}{12t} \right) - w \left( \frac{1}{4} - \frac{\delta + w}{12t} \right)
\]  

(36)

These formulas illustrate the trade-offs the two operators face in \( w \). An increase in \( w \) would lead to a general increase of the retail price level. This price increase again increases the profits of the Incumbent in the retail revenues from selling both, his lower as well as his higher value product. At the same time, an increase in \( w \) would decrease the sold quantity of his own lower value product as well as the sold wholesale quantity. Therefore, the Incumbent faces a trade-off between an increase in retail revenues in his fibre business segment and a decrease in his own copper network and his wholesale business.

The Entrant as well faces a trade-off between an increase of price and quantity in \( w \) of his higher value product: Just like the Incumbent, he profits in price and sold quantity in his fibre business yet he experiences a decrease in his sales in copper which again decreases his wholesale costs. The latter weakens the negative effect of this trade-off, making the trade-off of the Incumbent more binding than the one of the Entrant.

**Insight 4:**

Given the trade-off the incumbent faces in the copper access fee, he would not favour an arbitrarily high access fee.

Profit functions (35) and (36) can be summarised as follows:

\[
\pi_{I}^{sp} = \frac{\delta^2 + 18t^2}{24t} - \frac{w^2}{12t} - \frac{w(\delta - 6t)}{12t}
\]  

(37)

\[
\pi_{E}^{sp} = \frac{\delta^2 + 18t^2}{24t} + \frac{w^2}{24t} + \frac{w\delta}{12t}
\]  

(38)
They can be plotted as follows for the values of $\delta = 0.5, t = 0.5, v = 1$.

![Figure 5 Profit function of $I$](image1)

![Figure 6 Profit function of $E$](image2)

The sum of consumer surplus and the firms’ profits yields the welfare function:

$$WF = \delta^2 + \delta(\delta + 12t) - \frac{12t^2 - 48vt + w^2}{48t}$$

Insight 5:

Total welfare in case of coexistence of fibre and copper infrastructure is not independent of the level of the access fee $w$ as it was in the case of pure copper competition.

As opposed to the regulatory regime of the pure copper scenario described under 3.1, where the social planner could only determine an upper bound of the wholesale access fee of $w_{\text{max}} = t - v$ for the regulated access fee, he is now able to determine a concrete value that will optimise welfare in this scenario of coexistence of copper and fibre infrastructures. Also, the prior solution of maximum access fee that would keep the market existing, did not assure welfare optimal outcome and therefore only by “coincidence” the regulator may have set $w = \delta$, if $\delta$ laid in the named range of values.

3.2.5 Private optimum vs. social optimum

At this stage of the game, the social planner (sp) may decide on whether he shall refrain from setting the wholesale access fee and let the Incumbent determine this fee in the market equilibrium or whether he shall intervene with the market in order to maximise the total welfare. His second criterion for his decision making, besides welfare maximisation, will be the fibre take-up reached under each of the two possibilities.
Firstly, we shall assess the market equilibrium (me) outcomes where the Incumbent treats the wholesale access fee as one of his own prices. Solving the system of first order conditions of profit functions (4) and (5) under these conditions yields the following prices in market equilibrium of:

\[ p_{IH}^{me} = 3t - \frac{\delta}{4} \] (40)

\[ p_{IL}^{me} = 3t - \frac{3\delta}{4} \] (41)

\[ p_{EH}^{me} = \frac{9t}{4} \] (42)

\[ p_{EL}^{me} = \frac{15t}{4} - \delta \] (43)

Simultaneously, the outcome of the market equilibrium will yield the following wholesale access fee \( w^{me} \) that optimises the Incumbent's profit, given consumer behaviour and reactional prices of both firms:

\[ w^{me} = 3t - \delta \] (44)

Secondly, we shall assess the wholesale access fee under welfare optimisation that would be reached under the social planner.

Solving for the first order condition of the welfare function with respect to \( w \) yields the welfare optimal copper wholesale access fee:

\[ w^{sp} = \delta \] (45)

This means, in welfare optimum, the wholesale access fee of the copper network exactly corresponds to the incremental value consumers attribute to the higher quality product as opposed to the lower quality product.

**Insight 6:**

The welfare optimising wholesale access fee for copper corresponds to the incremental value consumers attribute to purchasing a fibre product as opposed to purchasing a copper product.

Comparing the wholesale access fee that would be reached under market equilibrium as a result of profit maximisation and the one reached under welfare optimisation shows an interesting insight: For \( t = \frac{2}{3} \delta \) the wholesale fee that the Incumbent sets equals the welfare optimising wholesale access fee of \( w^{sp} = \delta \). Consequently and somewhat surprisingly we find, that for constellations where \( t < \frac{2}{3} \delta \), i.e. where the incremental value the consumers attribute towards purchasing a fibre based product as opposed to a copper based one exceeds the transaction costs by far, the social planner would determine a wholesale access fee for the copper based product that will be higher than
the one the Incumbent would determine by himself. The reason for this lays in the described effects of quantities and his revenues. An increase of $w$ leads to a decrease of the revenues the Incumbent may generate with his wholesale business. As valuation for the fibre product is relatively high, the migration effect of the Entrant’s (as well as the Incumbent’s own) consumers towards the fibre infrastructure is being incentivised and therefore the effect of declining wholesale revenues even stronger. Therefore, the higher the incremental valuation of the fibre product of the consumers is, the higher the incentive would be for the Incumbent to counter steer such intrinsic consumer migration motivation by not let the copper access fee grow too high in order to counter steer this tendency by keeping the overall price level rather low.

**Insight 7:**

As the incremental perceived consumer valuation for the fibre based product as opposed to the copper based product is relatively higher than the transactional costs, the Incumbent has interest in a lower copper wholesale access fee than the social planner would intend on implementing.

For the comparison of welfare under private and social optimum we assume the case where $w^{sp} > w^{me}$.

Under private optimum, welfare yields the welfare of market equilibrium of:

$$WF^{me} = v - \frac{7t}{16} + \frac{3\delta}{4} + \frac{\delta^2}{16t} \quad (46)$$

Consequently, the welfare formula under social optimum yields:

$$WF^{sp} = v + \frac{\delta}{2} + \frac{7\delta^2}{48t} - \frac{t}{4} \quad (47)$$

The following graph shows for $t = 0.5$ the comparison of welfare under regulation scheme of welfare optimised wholesale access fee (45) (red) and under Incumbent-internalised wholesale price setting (44) (blue).
The graph shows that for $t = 0.5$ for all positive combinations of $\delta$ and $v$ ranging from $\{0..1\}$, the welfare under welfare optimised wholesale access charge is greater than the one Incumbent’s profit optimised. We therefore find it sufficiently proven that the social planner will be able to always assure that under regulation a higher total welfare would be achieved than it would result from leaving the decision on the wholesale access fee to the Incumbent.\(^7\)

### 3.2.6 Trade-off between fibre adoption and welfare

Consider now the case where the social planner, in addition to maximising welfare, intends to increase the adoption of the fibre networks as part of his regulatory agenda.

May the welfare optimal $w^{sp}$ be the starting point of further considerations for the social planner when being advised to increase fibre adoption, fibre adoption, as the sum of consumers having decided to purchase the high value products from either of the two frims yields:

$$q_H^{sp} = \frac{\delta}{6t} + \frac{6t + w}{12t}$$

\(^7\) With one exception where $t = \frac{2}{3} \delta$ where the market equilibrium will lead to exactly the welfare optimum.
At welfare optimal \( w^{SP} \), quantities (28) to (31) are as follows:

\[
q_{IH}^{SP} = \frac{1}{4} + \frac{\delta}{12t} \tag{48}
\]

\[
q_{IL}^{SP} = \frac{1}{4} - \frac{\delta}{12t} \tag{49}
\]

\[
q_{EH}^{SP} = \frac{1}{4} + \frac{\delta}{6t} \tag{50}
\]

\[
q_{EL}^{SP} = \frac{1}{4} - \frac{\delta}{6t} \tag{51}
\]

For the purpose of illustration, let’s assume \( t = \delta = 0.5 \), then the consumers allocate as follows: \( q_{IH}^{SP} = 0.33, q_{IL}^{SP} = 0.17, q_{EH}^{SP} = 0.42 \) and \( q_{EL}^{SP} = 0.08 \), resulting in a total fibre adoption rate of 0.75 or 75% as the social planner chose the wholesale access fee that maximised welfare, given firms’ pricing decisions. At given parametrisation it shows that the welfare optimum outcome is none where all consumers allocate on the higher value products.

Graphically, both graphs of welfare and total fibre adoption at the location \( t = \delta = 0.5 \) and \( v = 1 \) show as follows:

![Graph showing welfare and fibre adoption functions in dependence on \( w \) at \( t = 0.5, \delta = 0.5, v = 1 \)]

Figure 8 Welfare and fibre adoption functions in dependence on \( w \) at \( t = 0.5, \delta = 0.5, v = 1 \)

This graph show that to a certain extent, welfare increases with fibre adoption in \( w \) up to the point where \( w^{SP} = \delta \). Beyond this maximum, welfare declines in \( w \) while fibre adoption continues to increase. Therefore, an increase in fibre adoption beyond the point of \( w^{SP} = \delta \) will result in a decrease of welfare and an increase in fibre adoption. As
we have discussed in 3.2.3, a certain amount of consumers will not be able to be motivated to switch to the fibre infrastructure by steering the wholesale access fee \( w \). Therefore, beyond the point where all Entrant copper clients have been induced to switch to either of the three alternatives, at the costs of welfare loss that may come along with this, a further increase in \( w \), will only lead to a further decrease of welfare yet not anymore to further increase of fibre adoption.

Insight 8:

Total welfare and fibre adoption both increase only to a certain extent after which fibre adoption continues to climb, while welfare declines. Not all consumers are migratable to fibre in steering the copper access fee.

At the chosen example of values of at \( t = 0.5, \delta = 0.5, \nu = 1 \), welfare would be 1.2 in the point of welfare optimal wholesale access fee. We may compare this welfare value and total fibre adoption of 75% to the welfare that would be reached in the same example of parameters if the Incumbent was to choose the wholesale access fee of \( w^{me} = 3t - \delta \). Under such conditions, total fibre adoption would increase by 11% from 75% to 83% while welfare would decrease to 1.19, i.e. by 0.9%. Consequently, in case the social planner may not be satisfied by the fibre adoption reached under welfare optimisation, he may choose to increase fibre adoption in return for a decrease in welfare. He would have the tools at hand to weigh these two counter steering effects. Therefore, despite fibre adoption under market equilibrium being higher than under social optimum (under such constellations where the social planner would choose a lower copper access fee than the incumbent), his ability of caring for a overall higher welfare as well as his awareness over the trade-off in fibre adoption and welfare and being able to act correspondingly, we see it as reasonable to conclude that the social planner should remain in charge of regulating the access fee to the copper network.

4 Conclusion

We firstly analyse competition under a baseline scenario of sole copper competition and prove that the wholesale access fee is passed on symmetrically by both operators to the end customers in prices. Given that total demand is fixed in equilibrium, this leads to a total welfare that is neutral with respect to the wholesale access fee leaving the social planner no means to optimise welfare. Consequently, in the second analysed scenario of coexistence of the copper and fibre products, the social planner will have to reassess market conditions. When analysing operators’ pricing strategies under coexistence, we show that the copper wholesale access fee will still be passed on to end customers but that both operators choose different pricing strategies: While the Incumbent applies the price increase induced by an access fee symmetrically on both of his products, the Entrant does so asymmetrically by increasing his copper prices more than his fibre prices. He thereby decreases his wholesale costs as consumers see incentives to
switch from the Entrant’s copper product to other products while simultaneously making his fibre products more attractive and inducing demand shifts towards it.

Deriving operators’ profits given demand shifts induced by these asymmetric pricing strategies, we find that both operators experience trade-offs in the wholesale access fees, with the trade-off of the Incumbent being more binding: As the wholesale access fee increases, the Incumbent loses an increasing number of customers to his competitor’s fibre product. We therefore can conclude that the Incumbent would not favour the wholesale access fee to be arbitrarily high even if he were to decide on it.

As the social planner faces the decision of whether to implement a welfare optimal wholesale access fee or to let the wholesale access fee be endogenously determined by the Incumbent as the result of profit maximisation, we develop conditions under which the Incumbent would choose even a smaller fee than the social planner. Such conditions are satisfied if consumers’ valuation of the fibre product as opposed to the copper one is significantly higher than their transportation costs. Applying this to a real world context, we think this finding may explain why an Incumbent chooses to deploy point-to-multipoint fibre infrastructures rather than point-to-point, as consumer experience should be lower this way.

Furthermore, we characterise the relation of fibre take-up and welfare by finding out that fibre take-up and welfare both increase simultaneously in the wholesale access fee up to a critical threshold. Beyond this threshold, additional take-up will be paid by loss of total welfare. Also we find that a complete adoption of the fibre infrastructure will never be inducible by steering the wholesale access fee as some fraction of consumer demand is independent of the access fee and hence will always stick to the Incumbent’s copper product. An increase of the wholesale access fee in the limit would therefore only decrease welfare but not lead to any further fibre adoption.

Our findings are relevant for political decision-making on promoting fibre adoption by the means of the copper wholesale fee. We inform about the limitations of such a measure and characterise the trade-offs for fibre adoption in different access fee scenarios.

5 Outlook

We have derived the presented results in a scenario of full presence of a copper as well as two competing fibre infrastructures. While this scenario may hold for densely populated areas in Europe or to a futuristic scenario, we would like to expand our analysis to scenarios of varying availabilities among infrastructure and operating firm. Such expansion of the applied model would allow us to derive more general results. Such generality could be achieved by dividing the total population into subpopulations.
that differ by four categorisations of availability of infrastructure as depicted in below figure of model design:

In this model subpopulations would be the following: Subpopulation I would be the underlying of the present model, where copper and fibre products are available to the population by two firms. In subpopulation II and III copper based products would be available by both firms while only one of two operators will have rolled out a fibre infrastructure in the footprint of the respective subpopulation. Lastly, subpopulation IV will only have the two copper products to their availability.

Such analysis would allow us to make our findings more robust in terms of more general assumptions of availability of infrastructures.
ANNEX

ANNEX I

Consumer surplus for consumers deciding to buy their copper-based internet product at the Incumbent is derived as follows:

\[ CS_{IL} = \int_{0}^{q_{IL}} \left( v + \delta_{IL} - tx - p_{IL} \right) dx \]

\[ \Leftrightarrow CS_{IL} = \frac{v}{2} - \frac{5t}{8} - \frac{w}{2} \]

Correspondingly, the consumer surplus for the Entrant’s customers ends in:

\[ CS_{EL} = \int_{0}^{q_{EL}} \left( v + \delta_{EL} - tx - p_{EL} \right) dx \]

\[ \Leftrightarrow CS_{EL} = \frac{v}{2} - \frac{5t}{8} - \frac{w}{2} \]

ANNEX II

Locations of the indifferent consumers are as follows:

Location of the indifferent consumer with first preference \( IH \):

\[ x_{IH/IL} = \frac{1}{2} + \frac{\delta}{4t} \]

\[ x_{IH/EH} = \frac{1}{2} - \frac{w}{8t} \]

\[ x_{IH/EL} = \frac{1}{2} + \frac{2\delta + w}{8t} \]

Location of the indifferent consumer with first preference \( IL \):

\[ x_{IL/IH} = \frac{1}{2} - \frac{\delta}{4t} \]

\[ x_{IL/EL} = \frac{1}{2} + \frac{w}{8t} \]

\[ x_{IL/EH} = \frac{1}{2} - \frac{2\delta + w}{8t} \]

Location of the indifferent consumer with first preference \( EL \):
\[ x_{EL/EH} = \frac{1}{2} - \frac{\delta + w}{4t} \]
\[ x_{EL/IL} = \frac{1}{2} - \frac{w}{8t} \]
\[ x_{EL/IH} = \frac{1}{2} - \frac{2\delta + w}{8t} \]

Location of the indifferent consumer with first preference \( EH \):
\[ x_{EH/BL} = \frac{1}{2} + \frac{\delta + w}{4t} \]
\[ x_{EH/IH} = \frac{1}{2} + \frac{w}{8t} \]
\[ x_{EH/IL} = \frac{1}{2} + \frac{2\delta + w}{8t} \]

**ANNEX III**

Consumer surplus functions of the four spokes:

\[ CS_{IH} = \frac{5\delta}{96t} - \frac{w^2}{384t} - \frac{(3\delta+8t)w}{64t} + \frac{(t+4v)\delta}{48t} - \frac{7t}{16} + \frac{v}{4} \]
\[ CS_{IL} = -\frac{\delta^2}{32t} - \frac{w^2}{384t} - \frac{(7\delta-24t)w}{192t} + \frac{(11t-4v)\delta}{48t} - \frac{7t}{16} + \frac{v}{4} \]
\[ CS_{BL} = -\frac{\delta^2}{32t} + \frac{7w^2}{128t} + \frac{(5\delta-4t-16v)w}{192t} + \frac{(11t-4v)\delta}{48t} - \frac{7t}{16} + \frac{v}{4} \]
\[ CS_{EH} = \frac{5\delta^2}{96t} - \frac{11w^2}{384t} + \frac{(5\delta-4t+16v)w}{192t} + \frac{(t+4v)\delta}{48t} - \frac{7t}{16} + \frac{v}{4} \]
References


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